

Seat No:

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A Charutar Vidya Mandal's Institution
V. P. & R. P. T. P. SCIENCE COLLEGE
(AUTONOMUS) (Grant-in-Aid)



Affiliated to Sardar Patel University, Vallabh Vidyanagar -388120

Bachelor of Science (Semester 2) Internal Examination-2025

US02MAMTH01:Algebra

Date: 04-03-2025 (Tuesday)

Time: 12:45 pm to 2:15 pm

Total marks: 30

NOTE: The symbols used in this paper have usual meaning unless specified.

Q. 1 Write the most appropriate answer from the following multiple-choice questions. [06]

- (1) For $\bar{z} = -1 + i$ then $\frac{1}{z}$ is ...
(a) $\frac{1}{2}(-1 - i)$ (b) $\frac{1}{2}(1 + i)$ (c) $\frac{1}{2}(1 - i)$ (d) $\frac{1}{2}(-1 + i)$
- (2) If $z = -\sqrt{3} + i$, then the $\arg(z)$ is
(a) $\frac{\pi}{6}$ (b) $\frac{5\pi}{6}$ (c) $\frac{7\pi}{6}$ (d) $\frac{\pi}{3}$
- (3) For any two square matrices A and B if $AB = I$ then
(a) $BA = I$ (b) BA may not be equal to I (c) BA may not be defined (d) none of these
- (4) Rank of singular matrix of order 4 can be at most
(a) 1 (b) 2 (c) 3 (d) 4
- (5) Consider a system of n linear equations in m unknowns $AX = B$, where $\rho(A) = n$. Then it
(a) may be inconsistent (b) is always consistent
(c) consistent only if $\rho(A) = m$ (d) given information is insufficient
- (6) The eigen values of matrix A of order 3 are 1, 2 and x and its determinant is 10, then the value of x is....
(a) -5 (b) 10 (c) 5 (d) can not be determined

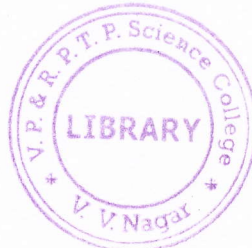
Q. 2 Prove that there are q and only q distinct values of $(\cos \theta + \sin \theta)^{\frac{1}{q}}$, where q is an integer. Use this to find cube root of unity. [06]

OR

Q. 2 Expand $\cos n\theta$ and $\sin n\theta$ in powers of $\cos \theta$, where $n \in \mathbb{N}$. Use this to expand $\cos n\theta$ in power of $\cos \theta$. [06]

Q. 3 (A) Show that the diagonal elements of skew-Hermitian matrices are either zero or purely imaginary. [02]

(B) Determine rank of the matrix $\begin{pmatrix} 1 & 2 & -1 & 0 \\ -1 & 3 & 0 & -4 \\ 2 & 1 & 3 & -2 \\ 1 & 1 & 1 & -1 \end{pmatrix}$ by reducing it to row echelon form. [04]



OR

Q. 3 Find inverse of the matrix $\begin{pmatrix} -1 & -3 & 3 & -1 \\ 1 & 1 & -1 & 0 \\ 2 & -5 & 2 & -3 \\ -1 & 1 & 1 & 0 \end{pmatrix}$ using elementary row operations, if exists. [06]

Q. 4 Solve the system of linear equations: [06]



$$x_1 + 2x_2 + 4x_3 + x_4 = 4,$$

$$2x_1 - x_3 - 3x_4 = 4,$$

$$x_1 - 2x_2 - x_3 = 0,$$

$$x_1 + x_2 - x_3 - 5x_4 = 5$$

OR

Q. 4 (A) Find reflection matrix for the reflection about the line $y = \tan \theta x$, $\theta \notin \{0, \pi, \pm \frac{\pi}{2}\}$. [03]

(B) Find rotation matrix that rotates any point through an angle θ in anti-clockwise direction. [03]

Q. 5 (A). Show that the characteristic roots of an idempotent matrix are either zero or unity. [03]

(B) Show that the two matrices A and $P^{-1}AP$ have the same Characteristic values. [03]

OR

Q. 5 Check whether the matrix $A = \begin{pmatrix} 1 & -6 & -4 \\ 0 & 4 & 2 \\ 0 & -6 & -3 \end{pmatrix}$ is diagonalizable or not? Justify your answer. [06]

Good Luck