



A Charutar Vidya Mandal's Institution
V.P. & R.P.T.P. SCIENCE COLLEGE

(AUTONOMOUS) (Grant-in-Aid)
Affiliated to Sardar Patel University
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Bachelor of Science (Semester - 1) Internal Examination - 2024
US01MAMTH01: Calculus

Date: 3rd September 2024, Thursday

Time: 11:00 AM to 12:30 PM

Total marks :30

Q.1 Select the correct option:

[06]

- 1) Let $f(x)$ be a real valued function defined on $[a, b]$. Suppose that $f(x)$ is differentiable on (a, b) , continuous on $[a, b]$ and $f(a)' = f(b)$, then there exist c in (a, b) such that
a) $f'(c) = \frac{f(b)-f(a)}{b-a}$ b) $f'(c) = 0$ c) $f'(c) = k$ d) None
- 2) Let $a, b \in R$. Then $y = \log(ax - b)$, then $y_n =$ ____
a) $\frac{n!a^n}{(ax-b)^n}$ b) $\frac{(-1)^n n! a^n}{(ax-b)^n}$ c) $\frac{(-1)^{n-1} (n-1)! a^n}{(ax-b)^{n-1}}$ d) $\frac{(-1)^{n-1} n! a^n}{(ax-b)^{n-1}}$
- 3) $1 + x + \frac{x^2}{2} + \frac{x^3}{3!} + \frac{x^4}{4!} + \dots$ is the power series expansion of which function.....
a) $\cos x$ b) $\sin x$ c) $\frac{1}{1+x}$ d) e^x
- 4) $f(x, y) = x^2 y^4 - x^3 y^3 + x y^5$ is a homogeneous function of degree ____
a) 0 b) 6 c) 5 d) 4
- 5) If $u = x^3 - 3xy^2$, then $\frac{\partial^2 u}{\partial x \partial y} =$ _____.
a) $-6y$ b) $6y$ c) $6x$ d) $-6x$
- 6) Asymptotes of $y = x^3 - 3x^2 - 2x$ are.....
a) $x = 0, 1, 2; y = 1$ b) $x = 0, -1, 2; y = 1$
c) $x = 0, -1, 2$ d) Not possible

Q.2 State and prove Leibniz's theorem. Use this theorem to find y_n for $y = x \log(x - 1)$. [06]

OR

Q.2 State and prove Lagrange's Mean Value theorem. Examine the validity of the hypothesis [06]
of this theorem for $f(x) = \log x$ for $\left[\frac{1}{2}, 2\right]$.

Q.3 State Taylor's theorem with Lagrange's form of remainder. And use this theorem to obtain [06]

Taylor's series expansion for $f(x) = \log(1+x)$ for $|x| < 1$.

OR

Q.3 Obtain reduction formula for $\int \cos^m x \, dx$. [06]

OR

Q.3 Evaluate $\lim_{x \rightarrow 0} \frac{e^x + \log(1-x) - 1}{\tan x - x}$ (Use L-hospital Rule) [06]

Q.4 If a curve given by $x = x(t)$, $y = y(t)$ and both x and y get numerically large as t approaches some number say a . Then an oblique asymptotes to the curve if it is exist, and given by $y = mx + c$, where $m = \lim_{t \rightarrow a} \frac{dy}{dx}$, $c = \lim_{t \rightarrow a} (y - mx)$. [06]

OR

Q.4 Sketch the curve $r = 3(1 + \cos\theta)$. [06]

Q.5 If $u = f(x^2 + y^2)$, where $x = r \cos \theta$, $y = r \sin \theta$, then show that [06]

$$\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial u}{\partial y}\right)^2 = (f'(r))^2$$

OR

Q.5 State & prove Euler's theorem for $z = f(x, y)$. Verify this theorem for function [06]

$$z = \sin^{-1}\left(\frac{x}{y}\right) + \tan^{-1}\left(\frac{y}{x}\right).$$



Best of Luck