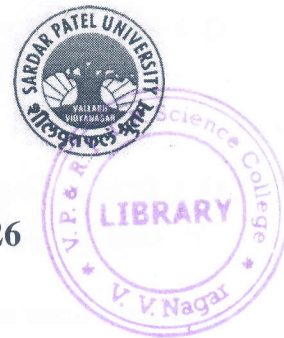




A Charutar Vidya Mandal's Institution
V. P. & R. P. T. P. SCIENCE COLLEGE
 (AUTONOMUS) (Grant-in-Aid)

Affiliated to Sardar Patel University, Vallabh Vidyanagar -388120

Bachelor of Science (Semester VI) Internal Examination-2026
US06MAMTH02:Linear Algebra



Date: 07-03-2026 (Saturday)

Time: 10:45 am to 12:15 pm

Total marks: 30

NOTE: The symbols used in this paper have usual meaning unless specified.

Q. 1 Write the most appropriate answer.

[06]

- (1) The dimension of subspace $W = \{(x, y, z, w) \in \mathbb{R}^4 : x + y + 2w = 0\}$ of vector space V is
 (a) 1 (b) 2 (c) 0 (d) 3
- (2) What is the span of $\{(1, 2), (2, 4)\}$?
 (a) $\mathbb{R}^2$ (b) $\mathbb{R}$ (c) $y = x$ line (d) None
- (3) Which of the following is a linear transformation on $\mathbb{R}$?
 (a) $T(x) = x^2$. (b) $T(x) = \cos(x)$. (c) $T(x) = -2x$. (d) $T(x) = x - 1$.
- (4) For a linear map $T : \mathbb{R}^2 \rightarrow \mathbb{R}^2$, the dimension of null space is 1, then what is the dimension of Range space?
 (a) 0 (b) 1 (c) 2 (d) Not possible
- (5) For a linear transformation $T : \mathbb{R}^4 \rightarrow \mathbb{R}^3$, the order of associated matrix with T is
 (a) 3×4 (b) 4×3 (c) 3×3 (d) 4×4
- (6) If $u = (1, 2, 3)$ and $v = (0, 1, -3)$ then $\|u + v\|^2 = \dots$
 (a) 10 (b) 24 (c) 14 (d) 4

Q. 2 Let $\mathbb{R}^+$ be the set of all positive real numbers. Define the operations as below.

$$u + v = uv, \quad \forall u, v \in \mathbb{R}^+ \quad \text{and} \quad \alpha u = u^\alpha, \quad \forall \alpha \in \mathbb{R}, u \in \mathbb{R}^+$$

Prove that $\mathbb{R}^+$ is a real vector space.

[06]

OR

Q. 2 (i) Let V be a vector space, then prove that, the set v_1, v_2, v_3 is L.D. iff v_1, v_2, v_3 are coplanar. [04]

(ii) Let A and B be two vector spaces, then show that $A \cup B$ is not a vector space. [02]

Q. 3 For a linear map $T : \mathbb{R}^2 \rightarrow \mathbb{R}^2$, $T(1, 2) = (1, 0)$ & $T(1, 1) = (3, 2)$. Find the formula for $T(x, y)$, where $(x, y) \in \mathbb{R}^2$. [06]

OR

Q. 3 Define a Linear Transformation. Prove that a mapping $T : U \rightarrow V$ is a linear transformation if and only if $T(k_1 u_1 + k_2 u_2) = k_1 T(u_1) + k_2 T(u_2)$, where k_1, k_2 are scalars and $u_1, u_2 \in U$. [06]

Q. 4 For a linear map $T : V_2 \rightarrow V_2$, defined by $T(x, y) = (2x - y, x + 2y)$. Find $T^{-1}(x, y)$. [06]

OR

Q. 4 For a matrix $A = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ and the bases B_1 and B_2 , given below, determine a linear map $T : V_3 \rightarrow V_3$ such that $A = (T : B_1, B_2)$, where $B_1 = \{(1, 1, 1), (1, 0, 0), (0, 1, 0)\}$ and $B_2 = \{(1, 2, 3), (1, -1, 1), (2, 1, 1)\}$. [06]

Q. 5 (A) Let V be an real inner product space. Then for any vectors $u, v \in V$, prove that $|u \cdot v| \leq ||u|| ||v||$. [06]

OR

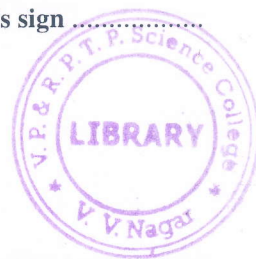
Q. 5 Orthonormalise the set of linearly independent vectors $\{(1, 0, 1, 1), (-1, 0, -1, 1), (0, -1, 1, 1)\}$ of V_4 . [06]



Good Luck

Seat No:

Supervisor's sign



Bachelor of Science (Semester VI) Quiz-2026
US06MAMTH02:Linear Algebra

Date: 07-03-2026 (Saturday)

Time: 20 minutes

Total marks: 15

NOTE: The symbols used in this paper have usual meaning unless specified.

Q. 1 Fill in the blanks with appropriate answer.

[08]

- (1) If $v_1, v_2, \dots, v_n$ are vectors in a vector space V , then $\text{Span}\{v_1, v_2, \dots, v_n\} = \{a_1v_1 + a_2v_2 + \dots + a_nv_n : a_i \in \mathbb{R}\}$ is called the _____ of the set $\{v_1, v_2, \dots, v_n\}$. [Span / Basis]
- (2) The set $\{v\}$ is linearly dependent $\iff v = \underline{\hspace{2cm}}$ [$\{\bar{0}\} / \{0\}$]
- (3) For a linear transformation $T : V \rightarrow W$, the set $\ker(T) = \{v \in V : T(v) = 0\}$ is called the _____ of T . [Kernel / Range]
- (4) If $\dim(V) = 5$ and $\text{rank}(T) = 3$, then $\text{nullity}(T) = \underline{\hspace{2cm}}$. [2 / 8]
- (5) If $A = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$, then $\text{rank}(A) = \underline{\hspace{2cm}}$. [2 / 1]
- (6) If A is an $m \times n$ matrix, then $\text{rank}(A) + \text{nullity}(A) = \underline{\hspace{2cm}}$. [n / m]
- (7) In the Gram-Schmidt process, the vectors obtained are _____. [Orthonormal / Linearly dependent]
- (8) In an inner product space, vectors u and v are orthogonal if $u \cdot v = \underline{\hspace{2cm}}$. [$0 / \bar{0}$]

Q. 2 Whether the following statement is true or false. Write your answer in the box.

[07]

- (1) For a vector $u \in V$, $0u = \bar{0}$. []
- (2) If a set of vectors is linearly independent, then one vector can be written as a linear combination of others. []
- (3) A linear transformation is non-singular if its kernel contains only the zero vector. []
- (4) The range of a linear transformation T is a subspace of W . []
- (5) Two matrices that represent the same linear transformation with respect to different bases may be same. []
- (6) For an inner product space V , $\|u + v\|^2 = \|u\|^2 + \|v\|^2$, $\forall u, v \in V$. []
- (7) In an orthonormal set, every vector has length 1 and vectors are mutually orthogonal. []

All The Best