



A Charutar Vidya Mandal's Institution
V. P. & R. P. T. P. SCIENCE COLLEGE
 (AUTONOMUS) (Grant-in-Aid)



Affiliated to Sardar Patel University, Vallabh Vidyanagar -388120

Bachelor of Science (Semester VI) Internal Examination-2026
US06MAMTH01:Real Analysis-2

Date: 06-03-2026 (Friday)

Time: 10:45 am to 12:15 pm

Total marks: 30

NOTE: The symbols used in this paper have usual meaning unless specified.

Q. 1 Write the most appropriate answer from the following multiple-choice questions. [06]

- (1) If a function f has a discontinuity of first type at $x = 2$ then $\lim_{x \rightarrow 2^-} f(x)$ and $\lim_{x \rightarrow 2^+} f(x)$ both
 - (a) do not exist.
 - (b) exist and they are equal.
 - (c) exist but they are not equal.
 - (d) cannot exist together.
- (2) If f is continuous on an interval $I = (a, b)$, $a, b \in \mathbb{R}$, then
 - (a) f is uniformly continuous on I .
 - (b) f is not necessarily uniformly continuous on I .
 - (c) f may have some points of discontinuities in I .
 - (d) f is not uniformly continuous on I .
- (3) The condition that a derivable function f monotonically increasing at c is
 - (a) $f'(c) = 0$.
 - (b) $f'(c) \neq 0$.
 - (c) $f'(c) \geq 0$.
 - (d) $f'(c) \leq 0$.
- (4) The series $\sum_{n=1}^{\infty} \frac{x^n}{n!}$
 - (a) is not convergent.
 - (b) converges only for $x = 0$.
 - (c) converges only for $x > 0$
 - (d) converges for every $x \in \mathbb{R}$
- (5) The sum of the series $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{(2n)!} \left(\frac{\pi}{2}\right)^{2n}$ is
 - (a) 1
 - (b) -1
 - (c) 0
 - (d) none of these
- (6) If sequence of continuous functions $\{f_n(x)\}$ converges uniformly to a function f on (a, b) and $c \in (a, b)$, then
 - (a) f may not be continuous at c
 - (b) f is continuous but $\lim_{n \rightarrow \infty} f_n(c) \neq \lim_{x \rightarrow c} f(x)$
 - (c) f is continuous but $\lim_{n \rightarrow \infty} f_n(c) = \lim_{x \rightarrow c} f(x)$
 - (d) none of these

Q. 2 If f and g are two functions defined on some neighbourhood of c such that $\lim_{x \rightarrow c} f(x) = l$ and $\lim_{x \rightarrow c} g(x) = m$, then prove that $\lim_{x \rightarrow c} (fg) = lm$. [06]

OR

Q. 2 Show that a continuous function on a closed and bounded interval $[a, b]$ attains its bounds at least once in $[a, b]$. [06]

Q. 3 State and prove Lagrange's Mean Value Theorem. [06]

OR

Q. 3 State and prove Darboux's theorem. [06]

Q. 4 For $x_1, x_2 \in \mathbb{R}$, prove that $C(x_1 + x_2) = C(x_1)C(x_2) - S(x_1)S(x_2)$. [06]

OR

Q. 4 Prove that there exists a positive number π such that $C\left(\frac{\pi}{2}\right) = 0$ and $C(x) > 0$ for $0 \leq x \leq \pi/2$. [06]

Q. 5 State and prove M_n test for uniform convergence of sequence of functions. [06]

OR

Q. 5 If a sequence $\{f_n\}$ converges uniformly to f in $[a, b]$ and c is a point of $[a, b]$ such that

$$\lim_{x \rightarrow c} f_n(x) = a_n, \quad n = 1, 2, 3, \dots$$

then prove that: (i) $\{a_n\}$ converges and (ii) $\lim_{x \rightarrow c} f(x) = a_n$. [06]



All The Best

Seat No:

Supervisor's sign



Bachelor of Science (Semester VI) Quiz-2026
US06MAMTH01:Real Analysis-2

Date: 06-03-2026 (Friday)

Time: 20 minutes

Total marks: 15

NOTE: The symbols used in this paper have usual meaning unless specified.

Q. 1 Whether the following statement is true or false. Write your answer in the box. [07]

- (1) If $|f|$ is continuous at a then f is also continuous at a . []
- (2) If $f : (a, b) \rightarrow \mathbb{R}$ is continuous at c and $f(c) > 0$ then $f > 0$ for all $x \in (a, b)$. []
- (3) Let $f : E \rightarrow \mathbb{R}$. If f is continuous at c , $c \in E$, then it is differentiable at c . []
- (4) Let $f : (a, b) \rightarrow \mathbb{R}$. If $f'(x) = 0$ for all $x \in (a, b)$ then f is a constant function. []
- (5) The cosine function is strictly increasing in $[0, \pi/2]$. []
- (6) The domain of Logarithmic function is $\mathbb{R}^+$. []
- (7) If $\sum f_n(x)$ converges uniformly on $[a, b]$, then $\int_a^b \sum f_n(x) dx = \sum \int_a^b f_n(x) dx$. []

Q. 2 Fill in the blanks with appropriate answer given in the bracket. [08]

- (1) If $\lim_{x \rightarrow a} f(x)$ exists but $f(a)$ does not exist then f possesses a discontinuity of
- (2) $\lim_{x \rightarrow 0} \frac{1}{1+e^{-\frac{1}{x}}} = \dots\dots\dots$
- (3) If $f(x) = |x|$ then $\lim_{x \rightarrow 0^-} f(x) = \dots\dots\dots$
- (4) Let $R_n = \frac{h^n(1-\theta)^{n-p}}{p(n-1)!} f^{(n)}(a + \theta h)$. Then for $p = \dots\dots\dots$, we get Cauchy's form of remainder in Taylor's theorem.
- (5) $E(x_1 + x_2) = \dots\dots\dots$
- (6) $\dots\dots\dots = 1 - \frac{x^3}{3!} + \frac{x^5}{5!} - \dots\dots$
- (7) For $0 < x < 1$, the sum of the series of functions $\sum_{n=1}^{\infty} x^n$ is
- (8) Every uniformly convergent series of functions is also convergent

All The Best