



A Charutar Vidya Mandal's Institution
V.P. & R.P.T.P. SCIENCE COLLEGE (AUTONOMOUS)
(Grant-in-Aid)
Affiliated to Sardar Patel University
VALLABH VIDYANAGAR-388120



Bachelor of Science (Semester 5) Internal Examination - 2025
US05MAMTH02 : Abstract Algebra

Date: 23/09/25

Time: 10:45am to 12:15pm

Total marks : 30

Q-1 Write the most appropriate answer from the following multiple-choice questions. [06]

- 1 Additive inverse of 5 in $\mathbb{Z}_{17}$ is
(a) 13 (b) 12
(c) -5 (d) 0
- 2 Every infinite cyclic group has Generators.
(a) 1 (b) odd
(c) 2 (d) infinite
- 3 Field has normal subgroup.
(a) 1 (b) 2
(c) 3 (d) none of above
- 4 $\text{Ch}(\mathbb{R})$ is
(a) 1 (b) ∞
(c) 0 (d) none of above
- 5 No. Of Maximal ideals of $\mathbb{Z}_{15}$ are.....
(a) 3 (b) 15
(c) 2 (d) 4
- 6 The associate element of $3 - i$ is
(a) $3 + i$ (b) $i - 3$
(c) $-3 - i$ (d) i



Q-2 Prove that $R = \left\{ \begin{bmatrix} a & b \\ c & 0 \end{bmatrix} / a, b, c, d \in \mathbb{R} \right\}$ Form a group under addition. (6Marks)

OR

Q-2 a) Prove that finite collection of subgroup of group G is also a subgroup of G. Give an example of it. (6Marks)



Q-3 a) Prove that every cyclic group is abelian. Is converse true?

(3Marks)

b) State and prove Fermat's theorem.

(3Marks)

OR

Q-3 Prove that kernel of group homomorphism is a subgroup and normal subgroup.

(6Marks)

Q-4 Every field is an Integral Domain. Discuss about converse.

(6Marks)

OR

Q-4 Let $f: R \rightarrow R'$ be any ring homomorphism then prove that f is one-one $\Leftrightarrow \ker f = \{0\}$.

(6Marks)

Q-5 Prove that an ideal M in $\mathbb{Z}$ is maximal ideal iff $M = p\mathbb{Z}$, for some prime no. p .

(6Marks)

OR

Q-5 State and prove first isomorphism theorem for rings.

(6Marks)



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BSC (Semester 5) Quiz - 2025
US05MAMTH02 : Abstract Algebra

Date: 23/09/25

Day: Tuesday

Time: 10:45am

Total marks :15

Q-1 Fill in the blanks with appropriate answer given in the bracket.

[08]

- (i) $(Z_{17}, \times_{17})$ form a..... { Simple Group, infinite group, infinite cyclic group }
- (ii) The set $\{1, -1, i, -i\}$ under complex multiplication forms:
{ A non-abelian group, A cyclic group, Not a group }
- (iii)is not an Integral Domain. $\{Z, \mu_{22}(R), Z_p\}$
- (iv) is generator of group Z_9 . $\{1, 3, 9\}$
- (v) Ring of gaussian integer is..... $\{Z, Z[\sqrt{-5}], Z[i]\}$
- (vi) Prime Ideals of Z is of the form.... $\{Z_m, mZ, pZ\}$
- (vii) $Z/pZ \cong$ $\{pZ, Z_p, z\}$
- (viii) $O[Z_{18}/\{\bar{0}, \bar{6}, \bar{12}\}] =$ $\{3, 6, 9\}$



Q-2 Whether the following statement is true or false. Write your answer in the box.

[07]

- (i) N, Z, Q, R are groups with Binary Operation +. []
- (ii) Klein 4 Group is abelian but not cyclic. []
- (iii) Set of all cosets form a group. []
- (iv) There are only two isomorphism of C onto C . []
- (v) Q is an Integral Domain. []
- (vi) Every subring is an Ideal. []
- (vii) Every prime ideal is not maximal ideal in Z . []