



A Charutar Vidya Mandal's Institution
V. P. & R. P. T. P. SCIENCE COLLEGE
 (AUTONOMUS) (Grant-in-Aid)



Affiliated to Sardar Patel University, Vallabh Vidyanagar -388120

Bachelor of Science (Semester V) Internal Examination-2025
US05MAMTH01:Real Analysis-I

Date: 22-09-2025 (Monday)

Time: 10:45 pm to 12:15 pm

Total marks: 30

NOTE: The symbols used in this paper have usual meaning unless specified.

Q. 1 Choose the most appropriate answer.

[06]

- (1) If a set $A \subset \mathbb{R}$ is bounded above then the set of all upper bounds of A is
 (a) bounded above. (b) bounded below. (c) bounded. (d) None of these.
- (2) For $x \in \mathbb{R}$, $-|x + 1| < 3$ is
 (a) true only if $-4 < x < 2$. (b) true only if $x > -4$.
 (c) true for all $x \in \mathbb{R}$. (d) not true for any $x \in \mathbb{R}$.
- (3) The infinite union of closed sets is
 (a) always closed. (b) may not be closed. (c) necessarily open. (d) None of these.
- (4) The set of limit points of a bounded sequence is
 (a) may be infinite. (b) always finite.
 (c) always infinite. (d) The given information is insufficient.
- (5) (i) $\forall M \in \mathbb{R}, \exists N \in \mathbb{N}$ such that $x_n > M, \forall n \geq N$.
 (ii) $\forall M \in \mathbb{R}, \exists n \in \mathbb{N}$ such that $x_n > M$.
 If $+\infty$ is a limit point of a sequence $\{x_n\}$ then
 (a) both (i) and (ii) are true. (b) (ii) is true but (i) may not be true.
 (c) both (i) and (ii) may not be true. (d) None of these.
- (6) The series $\sum_{n=2}^{\infty} \frac{1}{n(\log n)^p}$ is
 (a) convergent $\forall p \in \mathbb{R}$. (b) convergent if $p \leq 1$.
 (c) convergent if $p > 1$. (d) divergent $\forall p \in \mathbb{R}$.

Q. 2 Prove that every ordered set with the least upper bound property also has the greatest lower bound property. [06]

OR

Q. 2 (P) Prove that there is no rational number whose square is 3. [03]

(Q) Prove that supremum of the set $\{-\frac{1}{n} : n \in \mathbb{N}\}$ is 0. [03]

P.T.O.

Q. 3 State and prove Bolzano Weierstrass theorem for sets. [06]

OR

Q. 3 Prove that any subset of $\mathbb{R}$ is closed if and only if its complement is open. [06]

Q. 4 State and prove the Cauchy's general principle for convergence of a sequence. [06]

OR

Q. 4 (P) By definition prove that $\lim_{n \rightarrow \infty} \frac{3+2\sqrt{n}}{\sqrt{n}} = 2$. [03]

(Q) Prove that if $\{a_n\}$, $\{b_n\}$, and $\{c_n\}$ are real sequences such that $a_n \leq b_n \leq c_n$, for all n and if $\lim_{n \rightarrow \infty} a_n = \lim_{n \rightarrow \infty} c_n = l$ then $\lim_{n \rightarrow \infty} b_n = l$. [03]

Q. 5 (A) Test the convergence of the series $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}+\sqrt{n+1}}$. [02].

(B) State and prove Cauchy's root test. [04].

OR

Q. 5 (A) Test the convergence of the series $\sum_{n=1}^{\infty} \frac{n^2-1}{n^2+1} x^n$, $x > 0$. [04]

(B) State the Alternating Series test. [02]

All The Best

Seat No:

Supervisor's sign



Bachelor of Science (Semester V) Quiz-2025
US05MAMTH01:Real Analysis-I

Date: 22-09-2025 (Monday)

Time: 20 minutes

Total marks: 15

NOTE: The symbols used in this paper have usual meaning unless specified.

Q. 1 Whether the following statement is true or false. Write your answer in the box. [08]

- (1) There exists $x \in \mathbb{R}$ and $y > 0$ such that for all $n \in \mathbb{N}$, $n > xy$. []
- (2) For $x, y \in \mathbb{R}$, there exists an irrational number z such that $x < z < y$. []
- (3) If S and T are subsets of $\mathbb{R}$ then $(S \cap T)' = S' \cap T'$. []
- (4) There exists a subset of $\mathbb{R}$ which is both open and closed. []
- (5) There exists a convergent sequence having two different limit points. []
- (6) The sequence $\{(-1)^n (1 + \frac{1}{n})\}$ oscillates finitely. []
- (7) The series $\sum_{n=1}^{\infty} a_n$ is convergent if a_n tends to 0 as n tends to ∞ . []
- (8) Every absolutely convergent series is convergent. []

Q. 2 Fill in the blanks with appropriate answer. [07]

- (1) For any $x, y \in \mathbb{R}$, $|x - y| \geq \dots\dots\dots$
- (2) The infimum and supremum of the set $\{-2, -\frac{3}{2}, \dots, -\frac{n+1}{n}, \dots\}$ are $\dots\dots\dots$ and $\dots\dots\dots$, respectively.
- (3) The set of limit points of the sequence $\{1 \pm \frac{1}{n}\}_{n=2}^{\infty}$ is $\dots\dots\dots$
- (4) Interior of any subset of $\mathbb{R}$ is always a/an $\dots\dots\dots$ set.
- (5) $\lim_{n \rightarrow \infty} \frac{1}{n} (1 + 2^{1/2} + \dots + n^{1/n}) = \dots\dots\dots$
- (6) A necessary and sufficient condition for the convergence of monotonically increasing sequence is that it is $\dots\dots\dots$
- (7) For $0 < r < 1$, the series $\sum_{n=1}^{\infty} r^n$ is $\dots\dots\dots$

All The Best