



A Charutar Vidya Mandal's Institution
V. P. & R. P. T. P. SCIENCE COLLEGE
 (AUTONOMUS) (Grant-in-Aid)



Affiliated to Sardar Patel University, Vallabh Vidyanagar -388120

Bachelor of Science (Semester IV) Internal Examination-2026
US04MAMTH01:Mathematical Analysis-I

Date: 07-03-2026 (Saturday)

Time: 2:45 pm to 4:15 pm

Total marks: 30

NOTE: The symbols used in this paper have usual meaning unless specified.

Q. 1 Choose the most appropriate answer.

[06]

- (1) Which of the following condition/s implies the convergence of a sequence $\{x_n\}$ of real numbers
 - (a) Given $\epsilon > 0 \exists$ an $N \in \mathbb{N}$ such that for all $n \geq N$, $|x_{n+1} - x_n| < \epsilon$.
 - (b) Given $\epsilon > 0 \exists$ an $N \in \mathbb{N}$ such that for all $n \geq N$, $\frac{1}{(n+1)^2} |x_{n+1} - x_n| < \epsilon$.
 - (c) Given $\epsilon > 0 \exists$ an $N \in \mathbb{N}$ such that for all $n \geq N$, $(n+1)^2 |x_{n+1} - x_n| < \epsilon$.
 - (d) None of these.
- (2) Let $x_1 = \frac{5}{2}$. For $n \in \mathbb{N}$, let $x_{n+1} = \frac{1}{5}(x_n^2 + 6)$. Then, which of the following is TRUE?
 - (a) $\{x_n\}$ is an increasing sequence, and $\{x_n\}$ is NOT a bounded sequence.
 - (b) $\{x_n\}$ is NOT an increasing sequence, and $\{x_n\}$ is NOT a bounded sequence.
 - (c) $\{x_n\}$ is NOT a decreasing sequence, and $\{x_n\}$ is a bounded sequence.
 - (d) $\{x_n\}$ a decreasing sequence, and $\{x_n\}$ is a bounded sequence.
- (3) For $n \in \mathbb{N}$, let $x_n = (-1)^n \cos \frac{1}{n}$ and $y_n = \sum_{k=1}^n \frac{1}{n+k}$. Then which of the following is TRUE?
 - (a) $\sum_{n=1}^{\infty} x_n$ converges, and $\sum_{n=1}^{\infty} y_n$ does NOT converge.
 - (b) $\sum_{n=1}^{\infty} x_n$ does NOT converge, and $\sum_{n=1}^{\infty} y_n$ converges.
 - (c) $\sum_{n=1}^{\infty} x_n$ converges, and $\sum_{n=1}^{\infty} y_n$ converges.
 - (d) $\sum_{n=1}^{\infty} x_n$ does NOT converge, and $\sum_{n=1}^{\infty} y_n$ does NOT converge.
- (4) The sum of the infinite series $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{\pi^{2n+1}}{2^{2n+1}(2n)!}$ is equal to

(a) $-\pi$.

(b) $\frac{\pi}{4}$.

(c) $\frac{\pi}{2}$.

(d) $\frac{-\pi}{4}$.



P. T. O.

(5) Let $A \subset \mathbb{R}$ and $f : A \rightarrow \mathbb{R}$ has a limit at $c \in \mathbb{R}$, then

- (a) f is bounded on A .
- (b) f is unbounded on A .
- (c) f is bounded in a neighborhood of c .
- (d) f may not be bounded in a neighborhood of c .

(6) Which of the following statement is NOT true.

- (a) There exist a function on $\mathbb{R}$, which is nowhere continuous.
- (b) There exist a function on $\mathbb{R}$, which is continuous only at 0.
- (c) There exist a function on $\mathbb{R}$, which is continuous only on Q .
- (d) There exist a function on $\mathbb{R}$, which is continuous only on Q^c .

Q. 2

[06]

(A) Prove that $\lim_{n \rightarrow \infty} \frac{1}{n+1} = 0$.

(B) Prove that a convergent sequence of real numbers is bounded.

OR

Q. 2 State and prove Monotone Subsequence theorem.

[06]

Q. 3 State and prove limit comparison test.

[06]

OR

Q. 3 State and prove Cauchy's condensation test.

[06]

Q. 4 Suppose

(a) the partial sums A_n of $\sum a_n$ form a bounded sequence;

(b) $b_0 \geq b_1 \geq b_2 \geq \dots$;

(c) $\lim_{n \rightarrow \infty} b_n = 0$.

Then prove that $\sum a_n b_n$ converges.

[06]

OR

Q. 4 State and prove sequential criteria for existence of limit of a function.

[06]

Q. 5 State and prove that maximum-minimum theorem.

[06].

OR

Q. 5 State and prove Bolzano's Intermediate Value Theorem.

[06]



All The Best

Bachelor of Science (Semester IV) Quiz-2026
US04MAMTH01:Mathematical Analysis-I

Date: 07-03-2026 (Saturday)

Time: 20 minutes

Total marks: 15

NOTE: The symbols used in this paper have usual meaning unless specified.

Q. 1 Whether the following statement is true or false. Write your answer in the box. [07]

- (1) There exists a non-negative divergent sequence $\{x_n\}$ such that $a \leq x_n \leq b$, for some $a, b \in \mathbb{R}$. []
- (2) There exists a convergent sequence $\{x_n\}$ with $\limsup x_n = 1$ and $\liminf x_n = 0$. []
- (3) The series $\sum_{n=1}^{\infty} a_n$ is convergent if a_n tends to 0 as n tends to ∞ . []
- (4) For a sequence $\{a_n\}$ of positive numbers $\liminf \frac{a_{n+1}}{a_n} \leq \liminf (a_n)^{1/n}$. []
- (5) The set of cluster points of the interval $(0, 1)$ is $[0, 1]$. []
- (6) If the series $\sum a_n$ is convergent then its every rearrangement is convergent. []
- (7) If f is continuous on a closed and bounded interval $[a, b]$ then $f([a, b]) = [f(a), f(b)]$. []

Q. 2 Fill in the blanks with appropriate answer. [08]

- (1) A necessary and sufficient condition for the convergence of monotonically increasing sequence is that it is
- (2) The set of sub-sequential limits of the sequence $\{(-1)^n\}_{n=2}^{\infty}$ is
- (3) For $0 < r < 1$, the series $\sum_{n=1}^{\infty} r^n$ is
- (4) $\lim_{n \rightarrow \infty} \frac{n^{-2026}}{(1.001)^n} = \dots\dots\dots$
- (5) Every absolutely convergent series is convergent
- (6) Let $p : \mathbb{R} \rightarrow \mathbb{R}$ be the polynomial function

$$p(x) = \frac{1}{2^{100}} x^{100} - 2^{99} x^{99} - 2^{98} x^{98} \dots - 2x - 1.$$

Then $\lim_{x \rightarrow \infty} p = \dots\dots\dots$

- (7) A continuous function f on closed and bounded interval I attains its
- (8) A uniformly continuous function maps a Cauchy sequence to

All The Best

