



A Charutar Vidya Mandal's Institution
V. P. & R. P. T. P. SCIENCE COLLEGE
 (AUTONOMUS) (Grant-in-Aid)



Affiliated to Sardar Patel University, Vallabh Vidyanagar -388120

Bachelor of Science (Semester III) Internal Examination-2025
US03MAMTH01:Linear Algebra

Date: 23-09-2025 (Tuesday)

Time: 02:45 pm to 4:15 pm

Total marks: 30

NOTE: The symbols used in this paper have usual meaning unless specified.

Q. 1 Write the most appropriate answer.

[06]

- (1) Which of the following is not a vector space over the field $\mathbb{R}$?
 - (a) Set of all 2×2 real matrices under usual addition and scalar multiplication.
 - (b) Set of all real-valued functions under usual addition and scalar multiplication.
 - (c) Set of all positive real numbers under usual addition and scalar multiplication.
 - (d) $\mathbb{R}^3$ under usual addition and scalar multiplication.
- (2) The span of the set $\{(1, 2), (2, 4)\}$ in $\mathbb{R}^2$ is :
 - (a) a line.
 - (b) a plane.
 - (c) a point.
 - (d) All of $\mathbb{R}^2$.
- (3) Which of the following is a linear transformation on $\mathbb{R}$?
 - (a) $T(x) = x^2$.
 - (b) $T(x) = \sin(x)$.
 - (c) $T(x) = 2x$.
 - (d) $T(x) = 3x + 1$.
- (4) If a linear transformation $T : \mathbb{R}^4 \rightarrow \mathbb{R}^3$, has a rank 2 then its nullity is
 - (a) 1.
 - (b) 2.
 - (c) 3.
 - (d) 4.
- (5) The matrix representing reflection in the line $y = x$ is :
 - (a) $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$
 - (b) $\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$
 - (c) $\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$
 - (d) $\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$
- (6) A real symmetric matrix is :
 - (a) always non-diagonalizable
 - (b) diagonalizable only if it is positive definite.
 - (c) diagonalizable only if it has distinct eigenvalues.
 - (d) always orthogonally diagonalizable.

Q. 2 (A) Let S be a non-empty subset of a vector space V and $u, v \in V$. If $u \in [S \cup \{v\}]$, but $u \notin [S]$, then prove that $v \in [S \cup \{u\}]$. [03]

(B) Check whether the set $\{(1, 2, 1), (-1, 1, 0), (5, -1, 2)\}$ is basis for $\mathbb{R}^3$ or not? [03]

OR

Q. 2 (P) Define direct sum of vector spaces. Show that $\mathbb{R}^2 = W_1 \oplus W_2$, where $W_1 = \{(x, 0) : x \in \mathbb{R}\}$ and $W_2 = \{(0, y) : y \in \mathbb{R}\}$. [03]

(Q) Determine whether the set $W = \{(x, y, z) \in \mathbb{R}^3 : x + y + z = 0\}$ is a subspace of $\mathbb{R}^3$. [03]

- Q. 3 (A)** For the map $T : \mathbb{R}^2 \rightarrow \mathbb{R}^2$, $T(1, 2) = (1, 0)$ & $T(1, 1) = (3, 2)$. Find the formula for $T(x, y)$, where $(x, y) \in \mathbb{R}^2$.
- (B)** For the linear transformation $T : \mathbb{R}^3 \rightarrow \mathbb{R}^2$, $T(x, y, z) = (x + y, y + z)$, find the rank and nullity of T . [03]

OR

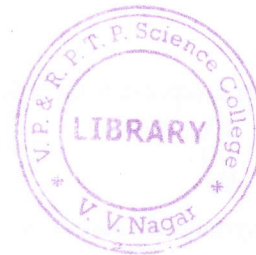
- Q. 3 (P)** Prove that a mapping $T : U \rightarrow V$ is a linear transformation if and only if $T(k_1u_1 + k_2u_2) = k_1T(u_1) + k_2T(u_2)$, where k_1, k_2 are scalars and $u_1, u_2 \in U$. [03]
- (Q)** Let $T_1, T_2 : \mathbb{R}^2 \rightarrow \mathbb{R}^2$ be defined by $T_1(x, y) = (x + y, y)$, $T_2(x, y) = (x, x - y)$. Find $(T_1 + T_2)(x, y)$ and $2T_1(x, y)$. Verify that both are linear maps. [03]
- Q. 4 (A)** Prove that dimension of the vector space $M_{m \times n}$ is mn . [03]
- (B)** Find T^{-1} for a map $T : P_2 \rightarrow \mathbb{R}^3$ defined by $T(a_0 + a_1x + a_2x^2) = (a_0, a_1, a_2)$. [03]

OR

- Q. 4 (P)** If y_1 and y_2 are solutions of $y'' + p(x)y' + q(x)y = 0$, then prove that the Wronskian $W[y_1(x), y_2(x)]$ satisfies the Abel's formula $W(x) = C e^{-\int p(x) dx}$, where C is a constant. [03]
- (Q)** For a map, $T : V_2 \rightarrow V_3$, defined by $T(x, y) = (x + y, 2x - y, 7y)$, find the matrix associated to it with respect to the standard basis B_1 and B_2 of V_2 and V_3 respectively. [03]
- Q. 5 (A)** State and prove the Cauchy–Schwartz inequality. [04]
- (B)** Apply the Gram–Schmidt process to orthonormalize $\{(1, 0, 0), (1, 1, 0)\}$ in $\mathbb{R}^3$. [02]

OR

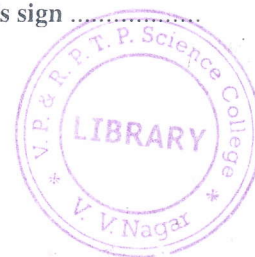
- Q. 5 (P)** Reduce the quadratic form $Q(x, y) = 3x^2 + 4xy + 3y^2$ to its canonical form by diagonalization of the associated symmetric matrix. [06]



Good Luck

Seat No:

Supervisor's sign



Bachelor of Science (Semester I) Quiz-2025
US03MAMTH01:Linear Algebra

Date: 23-09-2025 (Tuesday)

Time: 20 minutes

Total marks: 15

NOTE: The symbols used in this paper have usual meaning unless specified.

Q. 1 Whether the following statement is true or false. Write your answer in the box. [08]

- (1) For a vector $u \in V$, $0u = \bar{0}$. []
- (2) The span of a subset of a vector space is always a subspace. []
- (3) Every constant map from one vector space to another is both one-to-one and onto. []
- (4) If $T_2 \circ T_1$ is the zero map, then either T_1 or T_2 must be the zero map. []
- (5) If two solutions of a differential equation are linearly independent, their Wronskian is identically zero. []
- (6) Row operations do not change the rank of a matrix. []
- (7) For an inner product space V , $\|u + v\|^2 = \|u\|^2 + \|v\|^2$, $\forall u, v \in V$. []
- (8) $(\text{Row}(A))^\perp = \text{Nul}(A)$, where A is the matrix of size 2×3 . []

Q. 2 Fill in the blanks with appropriate answer. [07]

- (1) The dimension of the set of real valued functions defined and continuous on $[0, 1]$ (i.e., $C[0, 1]$) over $\mathbb{R}$ is [0,1, ∞]
- (2) The set $\{v\}$ is linearly dependent $\iff v = \dots\dots\dots$ [$\{0\}, \{1\}$]
- (3) Is $D^2 + D$ nilpotent on $V = P_2$, where D is Derivative operator and P_2 is set of polynomial of the degree less than or equal to 2. ? [.....] [Yes, No]
- (4) A linear map T is non-singular if and only if
- (5) The rank of a matrix plus its nullity equals the number of its [row, column]
- (6) For an inner product space V , $\forall u \in V$, $\|u\| = \dots\dots\dots$
- (7) Find the length of projection of $u = (1, 2)$ onto $v = (1, 1)$

All The Best