

Seat No:

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A Charutar Vidya Mandal's Institution
V. P. & R. P. T. P. SCIENCE COLLEGE
(AUTONOMUS) (Grant-in-Aid)



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Bachelor of Science (Semester 2) External Examination-2025
US02MAMTH01:Algebra

Date: 08-04-2025 (Tuesday)

Time: 02:00 pm to 4:30 pm

Total marks: 50

NOTE: The symbols used in this paper have usual meaning unless specified.

Q.1 Fill in the blanks.

[10]

- For $z = cis(\theta)$ and $\frac{1}{z} = cis(-\theta)$ then $z^{101101} - \frac{1}{z^{101101}} = \dots$
(a) $2i \sin(101101\theta)$ (b) $-2i \sin(101101\theta)$ (c) $2i \cos(101101\theta)$ (d) 0
- $z\bar{z} = \dots$
(a) z^2 (b) $|z|^2$ (c) $\bar{z}^2$ (d) None
- If $\begin{pmatrix} 4 & x+2 \\ 2x-3 & x+1 \end{pmatrix}$ is symmetric matrix then $x = \dots$
(a) -5 (b) 5 (c) -1 (d) 1
- If $A = \begin{pmatrix} 2x & 0 \\ x & x \end{pmatrix}$ and $A^{-1} = \begin{pmatrix} 1 & 0 \\ -1 & 2 \end{pmatrix}$ then $x = \dots$
(a) 1 (b) 2 (c) $1/2$ (d) None of these
- If $A = \begin{pmatrix} \cos(wx) & \sin(wx) & 0 \\ -\sin(wx) & \cos(wx) & 0 \\ 0 & 0 & 1 \end{pmatrix} = f(wx)$, then $A^{-1} = \dots$
(a) $f(x)$ (b) $f(wx)$ (c) $f(-x)$ (d) $f(-wx)$
- If A is an orthogonal matrix, then $|A| = \dots$
(a) -1 (b) 1 (c) 0 (d) ± 1
- $\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$ is in
(a) row-echelon form (b) reduced row echelon form (c) both (a) & (b) (d) None
- If $A = [a_{ij}]_{5 \times 5}$ and $a_{ij} = 4$, for $i \geq j$ and $a_{ij} = 0$, for $i < j$, then $\rho(A)$ is
(a) 5 (b) 0 (c) 4 (d) 3
- Which of the following is not true?
(a) If $k \neq 0$ is the eigen value of matrix A , then the eigen value of A^{-2} is $1/k^2$.

(b) If A is non-singular matrix then $(A^{-1})^{-1}$ is A .

(c) The matrix A has only one eigen vector corresponding to one eigen value.

(d) A square matrix A is invertible, if $\det(A) \neq 0$.

10. If a 3×3 matrix A has eigen (characteristic) values $0, -2, 6$ then the characteristic equation is

- (a) $x(x-2)(x+6)$ (b) $x(x+2)(x+6)$ (c) $x(x+2)(x-6)$ (d) $(x+2)(x-6)$

Q.2 (a) Show that the n^{th} roots of unity are in geometric progression and it's sum is zero. [05]

(b) Expand $\cos^8(\theta)$ in a series of multiple of θ . [05]

OR

Q.2 (a) State and prove De-Moivre's Theorem. [05]

(b) (i) Express $z = -\sqrt{3} + i$ in modulus and amplitude form. [05]

(ii) Find real and imaginary part of $z = (1+2i)(1-3i)$.

Q.3 (a) State and prove associative law for product of matrices. [05]

(b) Define rank of matrix. Determine rank of the matrix $\begin{pmatrix} 1 & 2 & 3 & 1 \\ 2 & 4 & 6 & 2 \\ 1 & 2 & 3 & 2 \end{pmatrix}$ by reducing it to row echelon form. [05]

OR

Q.3 (a) State and prove distributive law for matrices. [05]

(b) Using elementary operation, show that $\det \begin{pmatrix} a^2 & bc & ac+c^2 \\ a^2+ab & b^2 & ca \\ ab & b^2+bc & c^2 \end{pmatrix} = 4(abc)^2$. [05]

Q.4 (a) Define a row echelon form. Find the inverse of the matrix A by Gauss-Jordan method, where

$$A = \begin{pmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{pmatrix}. \quad [05]$$

(b) Explain, why in general (i) $(A \pm B)^2 \neq A^2 \pm 2AB + B^2$ (ii) $A^2 - B^2 \neq (A+B)(A-B)$. [05]

OR

Q.4 (a) (i) What are the possible solutions of homogeneous linear system of equations? [01]

(ii) Solve the following system of linear equations using Gaussian elimination method. [04]

$$x + 3y - 2z = 0, 2x - y + 4z = 0, x - 11y + 14z = 0$$

(b) Solve the system of linear equations by Cramer's rule:

[05]

$$x + 2y + 3z = 14; 3x + y + 2z = 11; 2x + 3y + z = 11$$

Q.5 (a) Verify the Cayley Hamilton theorem for the matrix $A = \begin{pmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{pmatrix}$. Use this theorem to find

$$A^{-1}$$

[05]

(b) Define skew Hermitian matrix. Prove that the modulus of characteristic root of a unitary matrix is unitary.

[05]

OR

Q.5 For the matrix $A = \begin{pmatrix} 3 & -1 & 1 \\ -1 & 5 & -1 \\ 1 & -1 & 3 \end{pmatrix}$.



[10]

(i) Find the eigen values and eigen vectors of the matrix A .

(ii) What is the algebraic multiplicity of each eigen value of the matrix A ?

(iii) When the matrix is diagonalizable? Is the given matrix A diagonalizable? Justify your answer.

Good Luck