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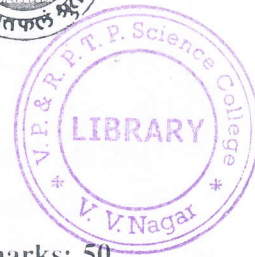


A Charutar Vidya Mandal's Institution
V. P. & R. P. T. P. SCIENCE COLLEGE
(AUTONOMUS) (Grant-in-Aid)

Affiliated to Sardar Patel University, Vallabh Vidyanagar -388120

Bachelor of Science (Semester 1) External Examination-2024

US01MAMTH01:Calculus



Date: 19-11-2024 (Tuesday)

Time: 11:00 am to 1:30 pm

Total marks: 50

NOTE: The symbols used in this paper have usual meaning unless specified.

Q. 1 Write the most appropriate answer from the following multiple-choice questions. [10]

- (1) If $y = -39x^3$ then $y_4 = \dots$
(a) 54834 (b) -54834 (c) 0 (d) None
- (2) Which of the following does not validate the hypotheses of Rolle's theorem?
(a) $f(x) = |x|$ on $[-2, 2]$ (b) $f(x) = \sin(\pi x)$ on $[-1, 1]$
(c) $f(x) = x(x-1)(x-2)$ on $[0, 2]$ (d) $f(x) = x^{\frac{2}{3}}$ on $[-1, 1]$
- (3) $x - \frac{x^3}{3!} + \frac{x^5}{5!} - \dots$ is the expansion of function.
(a) $f(x) = \cos x$ (b) $f(x) = \sin x$ (c) e^x (d) $\log(1-x)$
- (4) The value of $\lim_{x \rightarrow 0} \frac{\sin^{-1}(x)}{x}$ is
(a) 1 (b) 0 (c) -1 (d) None of these
- (5) If $u = e^{-x} \cos y$; $v = e^{-x} \sin y$ then $y = \dots$
(a) $\log(u^2 + v^2)$ (b) $\frac{1}{2} \log(u^2 + v^2)$ (c) $\tan^{-1}(\frac{v}{u})$ (d) $\tan^{-1}(\frac{u}{v})$
- (6) If $z = x^n \log(\frac{y}{x})$ then $y \frac{\partial z}{\partial y} = \dots$
(a) $\frac{x^n}{y}$ (b) $\frac{x^{n-1}}{y}$ (c) x^n (d) x^{n-1}
- (7) $r = \tan \theta \sec \theta$ represent a
(a) line (b) parabola (c) ellipse (d) circle
- (8) The polar form (r, θ) of the Cartesian form $(x, y) = (-\sqrt{3}, -1)$ is
(a) $(2, \frac{-\pi}{6})$ (b) $(2, \frac{-7\pi}{6})$ (c) $(2, \frac{7\pi}{6})$ (d) None
- (9) $\int_0^{\pi/2} \sin^{10}(x) dx = \dots$
(a) $\frac{63}{256}$ (b) $\frac{63}{512}$ (c) $\frac{63\pi}{512}$ (d) None
- (10) The horizontal asymptotes of $y = \frac{x^3}{x^2-1}$ is
(a) ± 1 (b) 1 (c) -1 (d) Not possible

Q. 2 (A) State and prove Leibniz's theorem. [05]

(B) Let $a, b, c \in \mathbb{R}$. For $y = e^{ax} \sin(bx + c)$, then prove that $y^n = r^n e^{ax} \sin(bx + c + n\phi)$,
where $r = \sqrt{a^2 + b^2}$, $\phi = \tan^{-1}(\frac{b}{a})$. [05]

OR

(1)

P.T.O

Q. 2 (A) State and prove Lagrange's Mean Value Theorem. [05]

(B) For $y = (x^2 - 2)^m$. Find the value of m such that $(x^2 - 2)y_{n+2} + 2xy_{n+1} - n(n+1)y_n = 0$. [05]

Q. 3 (A) Obtain Power series expansion of $f(x) = \log(1+x)$ for $|x| < 1$. [05]

(B) Evaluate $\lim_{x \rightarrow 0} \frac{e^x + \log(1-x) - 1}{\tan x - x}$. [05]

OR

Q. 3 (A) Obtain reduction formula for $\int \sin^n x$. [05]

(B) State and prove Cauchy's Mean Value theorem. [05]

Q. 4 (A) State and prove Euler's theorem for function of two variables. [05]

(B) If $z = f(x, y)$, where $x = r \cos \theta$, $y = r \sin \theta$, then prove that $(\frac{\partial z}{\partial x})^2 + (\frac{\partial z}{\partial y})^2 = (\frac{\partial z}{\partial r})^2 + \frac{1}{r^2}(\frac{\partial z}{\partial \theta})^2$. [05]

OR

Q. 4 (A) For $u = e^{xyz}$, find all third order partial derivatives. [05]

(B) If $u = \sin^{-1}(\frac{x^2 y^2}{x+y})$, then prove that $x^2 u_{xx} + 2xy u_{xy} + y^2 u_{yy} = 3 \tan u (3 \sec^2 u - 1)$. [05]

Q. 5 Discuss Intercepts, symmetry, Asymptotes and sign of function y for the curve $y = \frac{x^2-1}{x^2-4}$. Hence sketch the curve. [10]

OR

Q. 5 (A) Find an intrinsic equation of the curve $c : y = \cosh x, x \in R$ in the form $s = f(\psi)$ where s is measured from the point with co-ordinates $(0, 1)$ and ψ is the angle the tangent to c makes with the positive x -axis. [05]

(B) If the curve is given by $x = f(t), y = g(t)$ and both x and y get numerically large as t approaches some number say a then show that an oblique asymptotes to the curve if it exist is given by $y = mx + c$, where $m = \lim_{t \rightarrow a} \frac{dy}{dx}, c = \lim_{t \rightarrow a} (y - mx)$. [05]



Good Luck