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SARDAR PATEL UNIVERSITY
Bachelor of Science (Semester V) Examination - 2025
US05CMTH53: Group Theory

Date: 7/10/2025 (Tuesday)

Time: 10: 00 AM To 01:00 PM

Total: 70 Marks

Q-1 Answer the following by selecting the correct choice from the options 10

1. Order of i in multiplicative group $G = \{1, -1, i, -i\}$ is _____.
a) 0 b) 1 c) 2 d) 4
2. In the Klein 4- group $G = \{e, a, b, c\}$, inverse element of c is _____.
a) e b) a c) b d) c
3. The set $(GL(2, \mathbb{R}), \cdot)$ is _____.
a) a finite group b) an infinite group c) a cyclic group d) not a group
4. Cosets of a subgroup of a group are always _____.
a) disjoint b) same c) even d) odd
5. For Euler's ϕ - function, $\phi(p) =$ _____. (p is a prime number)
a) p b) $p-1$ c) $2p$ d) 1
6. If $H = m\mathbb{Z}$ is subgroup of additive group $G = \mathbb{Z}$ then $(G: H) =$ _____.
a) m b) $m+1$ c) $2m$ d) 1
7. Homomorphic image of a cyclic group is _____.
a) simple b) finite c) infinite d) cyclic
8. A homomorphism f is _____ iff $\ker f = \{e\}$.
a) onto b) one - one c) isomorphism d) automorphism
9. Order of S_4 is _____.
a) 12 b) 24 c) 60 d) 120
10. Signature of an odd permutation is _____.
a) -1 b) 2 c) 1 d) -2

Q-2 Answer any TEN

20

1. Prove that intersection of two subgroup of a group G is always a subgroup of the group G .
2. Find the order of every element of $(\mathbb{Z}_6, +)$.
3. Write any three subgroups of infinite cyclic group $\mathbb{Z}$.
4. Define Automorphism.
5. State Euler's theorem.

[P. T. O.]



6. Prove that any cyclic group of order n is isomorphic to Z_n .
7. Give two examples of simple groups.
8. If $\theta: G \rightarrow G'$ is a group homomorphism then show that $\ker \theta$ is a subgroup of G .
9. Define commutator subgroup.
10. Define signature of permutation.
11. If $H = K = Z_2$, find $Z_2 \times Z_2$.
12. Express the permutation $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 3 & 4 & 5 & 1 & 2 & 6 \end{pmatrix}$ as the product of disjoint cycles.

Q-3

- (a) Let H and K be subgroups of group G . Then prove that HK is a subgroup of G if and only if $HK = KH$. 5
- (b) Let G be a set with associative binary operation. Assume that G has a right unit element, and every element of G has a right inverse. Then prove that G is a group. 5

OR

Q-3

- (a) Let H and K are finite subgroups of group G such that HK is a subgroup of G . Then prove that $O(HK) = \frac{O(H)O(K)}{O(H \cap K)}$. 5
- (b) Let G be a group and $H = \{x \in G, xa = ax, \text{ for each } a \in G\}$. Then show that H is a subgroup of G . 5

Q-4

- (a) State and Prove Lagrange's theorem for a finite group. 5
- (b) Show that any non-cyclic group of order 4 is isomorphic to the Klein 4-group. 5

OR

Q-4

- (a) Let G be an infinite cyclic group. Then show that G has just one non-trivial automorphism. 5
- (b) Let G be a group and H be a subgroup of G . Show that the number (finite or infinite) of left cosets of H in G is the same as the number of right cosets of H in G . 5

Q-5

- (a) State and Prove first isomorphism theorem. 5
- (b) Let $\theta: G \rightarrow G'$ be a group homomorphism. If H is a subgroup of G then show that $H' = \theta(H)$ is a subgroup of G' . Also, show that if H' is a subgroup of G' then $H = \theta^{-1}(H')$ is a subgroup of G . 5

OR

Q-5

- (a) Let G be a group. Then show that G is abelian if and only if $i_x = I_G$, for all $x \in G$. 5
- (b) Let G be a group and G' the commutator subgroup of G . Then show that G/G' is abelian and if H is normal in G such that G/H is abelian then show that $G' \subset H$. 5

Q-6

- (a) Show that every $\sigma \in S_n$ can be expressed as a product of disjoint cycles. 5

- (b) State and Prove Caley's theorem for a group. 5

Q-6

- (a) Prove that the mapping $\epsilon: S_n \rightarrow \{1, -1\}$ given by $\sigma \rightarrow \epsilon \sigma$ is a homomorphism of the group S_n onto the multiplicative group of two elements $\{1, -1\}$. 5
- (b) Show that G is a direct product of subgroups H and K if and only if 5
- i) Every $x \in G$ can be uniquely expressed as $x = hk$, $h \in H$, $k \in K$.
 - ii) $hk = kh$, $h \in H$, $k \in K$.

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