



SARDAR PATEL UNIVERSITY

Bachelor of Science (Semester 5) Examination - 2025

US05CMTH52: Real Analysis-II

Date: 06/10/2025 (Monday)

Time: 10:00 AM to 01:00 PM

Total: 70 Marks

NOTE: Figure to the right indicate full marks of the questions.

Q-1 Answer the following by choosing correct answers from given choices.

[10]

- $\lim_{x \rightarrow 1} \frac{(x^2 - 1)}{x - 1} = \underline{\hspace{2cm}}$.
 (a) 0 (b) 2 (c) $\frac{1}{2}$ (d) 1
- The function $f(x) = x^2$ is uniformly continuous on _____.
 (a) $[-1, 1]$ (b) $(-1, 1)$ (c) $[-1, 1]$ (d) none of these
- $\lim_{n \rightarrow \infty} c_n = c \Leftrightarrow \lim_{n \rightarrow \infty} f(c_n) = f(c)$ iff a function $f: [a, b] \rightarrow R$ is _____.
 (a) Continuous (b) discontinuous (c) derivable (d) none of these
- $\log(1+x) = \underline{\hspace{2cm}}$ where $-1 \leq x \leq 1$.
 (a) $1 - x + \frac{x^2}{2} - \frac{x^3}{3} \dots$ (b) $x - \frac{x^3}{3!} + \frac{x^5}{5!} + \dots$ (c) $x - \frac{x^2}{2!} + \frac{x^3}{3!} + \dots$ (d) $x - \frac{x^2}{2} + \frac{x^3}{3} - \dots$
- For any $x, y \in [a, b]$ if $x > y$ then $f(x) > f(y)$ then f is _____ on $[a, b]$.
 (a) strictly decreasing (b) strictly increasing (c) decreasing (d) increasing
- What is Maclaurin's theorem?
 (a) Taylor's theorem with $a = 1$ (b) Taylor's theorem with $a = -1$
 (c) Taylor's theorem with $a = 0$ (d) none of these
- If $f(x, y) = \frac{xy}{x^2 + y^2}$ then $f_x(0, 0) = \underline{\hspace{2cm}}$.
 (a) 0 (b) 2 (c) 6 (d) 4
- $\lim_{(x, y) \rightarrow (0, 0)} \left(\frac{x \sin(x^2 + y^2)}{x^2 + y^2} \right) = \underline{\hspace{2cm}}$.
 (a) 1 (b) 3 (c) 0 (d) none of these
- If z is differentiable function of x, y and x, y are functions of u, v then
 $\frac{\partial z}{\partial u} = \underline{\hspace{2cm}}$.
 (a) $\frac{\partial z}{\partial x} \cdot \frac{\partial x}{\partial u}$ (b) $\frac{\partial z}{\partial y} \cdot \frac{\partial y}{\partial u}$ (c) $\frac{\partial z}{\partial x} \cdot \frac{\partial x}{\partial u} + \frac{\partial z}{\partial y} \cdot \frac{\partial y}{\partial u}$ (d) none of these
- A stationary point is called saddle point of a function f if it is _____ point.
 (a) extreme (b) non extreme (c) only maximum (d) none of these

Q-2 Answer TEN of the following.

[20]

- Define Left hand limit of a function.
- Show that the function $f(x) = x^2$ is derivable on $[0, 1]$.
- Define discontinuous function.
- Define decreasing function.
- State Lagrange's Mean value theorem.
- Define second form of Taylor's theorem.
- Define Explicit function.
- If $f(x, y) = 2x^2 - xy + 2y^2$ then find $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$ at the point $(1, 2)$.

[P. T. O.]

9. Prove that $\lim_{(x,y) \rightarrow (4,\pi)} x^2 \sin\left(\frac{y}{x}\right) = 8\sqrt{2}$.
10. State Maclaurin's theorem for function of two variables.
11. Define Maxima.
12. Show that $f(x, y) = y^2 + x^2y + x^4$ has minimum at $(0, 0)$.

Q-3

(A) Examine the continuity of $f(x) = \begin{cases} x e^{1/x} & x \neq 0 \\ 0 & x = 0 \end{cases}$. [5]

(B) Let f and g be the functions defined on some neighborhood. If $\lim_{x \rightarrow c} f(x) = l$ and [5]

$\lim_{x \rightarrow c} g(x) = m$ then show that $\lim_{x \rightarrow c} [f(x) - g(x)] = l - m$.

OR

(A) Show that a function which is derivable at a point is necessarily continuous at that point. [5]

(B) Prove that if a function f is continuous on a closed interval, then it is also uniformly continuous on that interval. [5]

Q-4

(A) State and prove Darboux's theorem. [5]

(B) Show that $\frac{\tan x}{x} > \frac{x}{\sin x}$; $0 < x < \frac{\pi}{2}$. [5]

OR

(A) State and prove Cauchy's Mean value theorem. [5]

(B) Prove that $\frac{v-u}{1+v^2} < \tan^{-1} v - \tan^{-1} u < \frac{v-u}{1+u^2}$. [5]

Q-5

(A) Show that $\lim_{(x,y) \rightarrow (0,0)} xy \left(\frac{\sqrt{x^2y^2+1}-1}{x^2+y^2} \right) = 0$. [5]

(B) State and prove Mean value theorem for two variables. [5]

OR

(A) Show that the repeated limit exists at the origin but simultaneous limit does not exist for $f(x, y) = \frac{x-y}{x+y}$. [5]

(B) Show that the function $f(x, y) = \begin{cases} xy \frac{x^2-y^2}{x^2+y^2} & x^2+y^2 \neq 0 \\ 0 & x=y=0 \end{cases}$ is differentiable at origin. [5]

Q-6

(A) Under the suitable conditions, show that $f(xy, z-2x) = 0$ satisfies the equation [5]

$x \frac{\partial z}{\partial x} - y \frac{\partial z}{\partial y} = 2x$.

(B) Find Maxima and Minima of the function $f(x, y) = x^3 + y^3 - 3x - 12y + 20$. [5]

OR

(A) Prove that, by the transformation $u = x - ct$, $v = x + ct$ the partial differential [5]

equation $\frac{\partial^2 z}{\partial t^2} = c^2 \frac{\partial^2 z}{\partial x^2}$ reduces to $\frac{\partial^2 z}{\partial u \partial v} = 0$.

(B) Expand $x^2y + 3y - 2$ in powers of $(x-1)$ and $(y+1)$. [5]

-----X-----X-----X-----

(.2)