



B.Sc. Sem : 5

Subject : Mathematics

Max. Marks : 70

Date: 04/10/2025

US05CMTH51 [ Real Analysis-1]

Time : 10.00 to 1.00

Q.1 Choose the correct option for each of the following.

[10]

(1) The field which does not have the least upper bound property is .....

- (a)  $Q$  (b)  $Z$  (c)  $N$  (d) None of these

(2) The greatest number of a set, if exists is .....

- (a) the supremum of the set (b) the infimum of the set (c) not unique (d) None of these

(3) The infimum of the set  $\left\{\frac{1}{n} / n \in N\right\}$  is .....

- (a) 0 (b) 1 (c) 2 (d) None of these

(4) Every open interval in  $R$  is .....set.

- (a) an open (b) a closed (c) open and closed (d) None of these

(5) The derived set of  $A = [a, b]$  is .....

- (a)  $\emptyset$  (b)  $R$  (c)  $(a, b)$  (d)  $[a, b]$

(6) The interior of the set  $Q$  is .....

- (a)  $N$  (b)  $R$  (c)  $\emptyset$  (d) None of these

(7) The Range of a sequence is always .....

- (a) empty (b) infinite (c) non-empty (d) None of these

(8) The range set of a sequence  $\{(-1)^n / n \in N\}$  is .....

- (a)  $\{-1\}$  (b)  $\{1\}$  (c)  $\{-1, 1\}$  (d) None of these

(9) A positive term series  $\sum \frac{1}{n^2}$  is .....

- (a) convergent (b) divergent (c) converges to 0 (d) None of these

(10) A series  $\sum u_n$  is convergent then  $\lim_{n \rightarrow \infty} u_n$ 

- (a)  $= 0$  (b)  $= 2$  (c)  $= 1$  (d) does not exist

Q.2 Attempt any TEN:

[20]

(1) Prove that the greatest lower bound of a set  $S$  is unique, if it exist.(2) Prove that  $\sqrt{3}$  is an irrational number.(3) Find the g.l.b and l.u.b of  $\left\{\frac{1}{m} + \frac{1}{n} / m, n \in N\right\}$  if they exist.

(4) Define : An open set.

(5) Define: A closed set

(6) Define: A limit point of a set .

(7) Define: Limit point of a sequence .

[1]

[P.T.O.]

(8) Prove that every convergent sequence is bounded.

(9) Prove that  $\lim_{n \rightarrow \infty} \frac{3+2\sqrt{n}}{\sqrt{n}} = 2$ .

(10) State Cauchy's General principle of convergence.

(11) Prove that the series  $\sum_{n=1}^{\infty} \frac{1}{n!}$  is convergent.

(12) Investigate the behaviour of the series  $\sum_{n=1}^{\infty} \frac{n}{n+1}$ .

Q.3 (a) State and Prove the Archimedean property of  $\mathbb{R}$ . [5]

(b) If  $x, y \in \mathbb{R}$  then prove that  $|x + y| \leq |x| + |y|$ . Also prove that  $||x| - |y|| \leq |x - y|$ . [5]

OR

Q.3 (a) Prove that the set of all rationals  $\mathbb{Q}$  is not an order complete field. [5]

(b) Prove that  $\sqrt{8}$  is an irrational number. [5]

Q.4 (a) State and Prove Bolzano-Weierstrass theorem for a set. [5]

(b) Prove that the arbitrary intersection of closed sets is closed. [5]

OR

Q.4 (a) Prove that a subset of  $\mathbb{R}$  is closed in  $\mathbb{R}$  if and only if its complement set is open in  $\mathbb{R}$ . [5]

(b) Let  $S \subset \mathbb{R}$  then prove that  $S^i$  is an open set. [5]

Q.5 (a) State and Prove Cauchy's first theorem on limits. [5]

(b) If  $\{a_n\}$  and  $\{b_n\}$  are two sequences such that  $\lim_{n \rightarrow \infty} a_n = a$  and  $\lim_{n \rightarrow \infty} b_n = b$  then prove [5]

that  $\lim_{n \rightarrow \infty} a_n b_n = ab$ .

OR

Q.5 (a) State and Prove Bolzano-Weierstrass theorem for a sequence. [5]

(b) Prove that  $\lim_{n \rightarrow \infty} \sqrt[n]{n} = 1$ . [5]

Q.6 (a) State and Prove comparison test of second type for a series. [5]

(b) Prove that the positive term geometric series  $1 + r + r^2 + \dots$  converges for  $r < 1$  and diverges to  $\infty$  for  $r \geq 1$ . [5]

OR

Q.6 (a) State and Prove Cauchy's Root Test for series. [5]

(b) Prove that the series  $\frac{1.2}{3^2 \cdot 4^2} + \frac{3.4}{5^2 \cdot 6^2} + \frac{5.6}{7^2 \cdot 8^2} + \dots$  converges. [5]



[2]