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SARDAR PATEL UNIVERSITY

Bachelor of Science (Semester 5) Examination – 2025
US05CMTH24 – Metric Spaces and Topological Spaces

Date: 08/10/2025

Time: 10:00 A.M. To 01:00 P.M.

Total: 70 Marks

Note: (R, u) : Set R with usual metric.

Q-1. Multiple Choice Questions

[10]

- (1) Which of the following condition is not in the definition of metric space (X, d) ?
- A. $d(x, y) \geq 0$ B. $d(x, y) = 0 \Leftrightarrow x = y$
C. $d(x, y) + d(y, z) \leq d(x, z)$ D. $d(x, y) = d(y, x)$
- (2) If ρ is a metric for a non-empty set M then how many new metrics can be made using ρ ?
- A. 1 B. 2
C. finite D. infinite
- (3) For which of the following set A , $\text{Int}(A)$ is open set in (R, u) ?
- A. $A = \{1, 2, 3, 4\}$ B. $A = (2, 5)$
C. $A = R$ D. All
- (4) For which of the following set, $\bar{A} = R$ in (R, u) ?
- A. $A = N$ B. $A = Z$
C. $A = \phi$ D. $A = Q$
- (5) The set $\{x \in R: x + 1 = 0\}$ is _____ in (R, u) .
- A. Open B. Closed
C. Dense D. Empty
- (6) Let X be any non-empty set. $\tau_1 = \text{Discrete topology}$, $\tau_2 = \text{Indiscrete topology}$. Then,
- A. τ_1 is finer than τ_2 B. τ_2 is finer than τ_1
C. Both are not comparable D. None of these
- (7) A is τ -open set iff
- A. $\bar{A} = A$ B. $X - A = A$
C. $\bar{\bar{A}} = A$ D. $\text{int}(A) = A$
- (8) Let $X = \{a, b, c\}$, $\tau = \{\phi, \{a\}, \{a, b\}, \{a, b, c\}\}$. Then which of the following set is closed in τ ?
- A. $\{b, c\}$ B. $\{a\}$
C. $\{a, b\}$ D. $\{d\}$
- (9) Which of the following is NOT always true?
- A. $A \subset \bar{A}$ B. $\text{Int}(A) \subset A$
C. $\text{Int}(A)$ is open set D. $\bar{A}$ is open set
- (10) Let A and B are two nonempty subsets of the topological space (X, τ) . Which of the following condition is there in the definition of disconnected space?
- A. $A \cap B = \phi$ B. $A \cup B = \phi$
C. $A \cap \bar{B} = \phi$ D. $\bar{A} \subset \bar{B}$



①

[P.T.O.]

Q-2. Short Answer Questions (Attempt any TEN)**[20]**

- (1) Define: Metric space.
- (2) For any metric space (X, d) , if we define $d_1(x, y) = \frac{d(x, y)}{1 + d(x, y)}$ then prove that d_1 satisfies the triangular inequality.
- (3) Define: Limit in metric space.
- (4) Give an example of cauchy sequence which is not convergent.
- (5) Define: Open set in metric space.
- (6) Prove that any finite subset of metric space is closed set.
- (7) Let $X = \{a, b, c\}$ and $\tau = \{\phi, \{a\}, \{a, b\}, X\}$. Then write all closed sets of (X, τ) .
- (8) Is lower limit topology finer topology than usual topology on $\mathbb{R}$? Justify your answer.
- (9) Define: Interior point in Topological Space
- (10) Prove that any space with more than one point with discrete topology is disconnected.
- (11) Find set of all cluster points of the set $\{x \in \mathbb{R} : |x| < 1\}$ in R with usual topology.
- (12) Define: Covering and Sub-covering.

- Q-3 (a)** Show that a sequence in a metric space cannot converge to two different points. **[05]**
(b) Let $\rho: R \times R \rightarrow R$ defined by $\rho(x, y) = |x - y|$. Then prove that ρ is **[05]**
Metric on R .

OR

- Q-3 (a)** Let $d: R \times R \rightarrow R$ be defined by $d(x, y) = \sin |x - y|$. Check whether d is a **[05]**
metric or not?
(b) Show that if d is metric for a non-empty set M then $5d$ is also metric on M . **[05]**

- Q-4 (a)** If f and g are continuous function from a metric space (M_1, ρ_1) into metric **[05]**
space (M_2, ρ_2) , then prove that $f - g$ is also continuous on M_1 .
(b) Let $f: R^2 \rightarrow R$ be a function defined by $f(x, y) = x, \forall x, y \in R^2$. Prove that f is **[05]**
continuous on R^2 .

OR

- Q-4 (a)** Prove that a subset G of the metric space (X, d) is open if and only if **[05]**
compliment of G is closed.
(b) Let (X, d) is metric space and let $A \subset B \subset M$. If A is dense in B and if B is **[05]**
dense in M , then prove that A is dense in M .
Q-5 (a) Prove that indiscrete topology satisfies all the conditions for becoming a **[05]**
topological space.
(b) Prove that arbitrary intersection of closed sets is also closed in topological space. **[05]**

OR

- Q-5 (a)** Prove that discrete topology satisfies all the conditions for becoming a **[05]**
topological space.
(b) Let (X, τ) be a topological space and A be a subset of X . A' is the set of all **[05]**
cluster points of A then prove that A is τ -closed if and only if $A' \subset A$.
Q-6 (a) Prove that continuous image of connected space is connected **[05]**
(b) Prove that if (X, τ) is disconnected then there is a non-empty proper subset of X **[05]**
which is both open and closed.

OR

- Q-6 (a)** Let $f: (R, \tau) \rightarrow (R, \psi)$ defined by $f(x) = 2, \forall x \in R$. Then prove that f is $\tau - \psi$ **[05]**
continuous.
(b) Prove that R with usual topology is connected space. **[05]**

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