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SARDAR PATEL UNIVERSITY
Bachelor of Science (Semester V) Examination - 2025
US05CMTH23: Group Theory

Date: 7/10/2025 (Tuesday)

Time: 10: 00 AM To 01:00 PM

Total: 70 Marks

Q-1 Answer the following by selecting the correct choice from the options

10

1. Multiplicative inverse of $\sqrt{5}$ in $\mathbb{R}^*$ is _____.
a) $-\sqrt{5}$ b) $\frac{1}{\sqrt{5}}$ c) 0 d) 1
2. In the Klein 4- group $G = \{e, a, b, c\}$, $ab =$ _____.
a) e b) a c) b d) c
3. The set $(M_2(\mathbb{R}), +)$ is _____.
a) a finite group b) an infinite group c) a cyclic group d) not a group
4. If H is subgroup of a group G with $O(G) = 10$, what can be the possible order of H ?
a) 3 b) 7 c) 5 d) 6
5. For Euler's ϕ - function, $\phi(11) =$ _____.
a) 1 b) 10 c) 9 d) 8
6. If $H = 7Z$ is subgroup of additive group $G = Z$ then $(G: H) =$ _____.
a) 5 b) 6 c) 2 d) 7
7. Homomorphic image of an abelian group is _____.
a) simple b) cyclic c) abelian d) finite
8. A homomorphism f is _____ iff $\ker f = \{e\}$
a) onto b) one - one c) isomorphism d) automorphism
9. Order of A_4 is _____.
a) 12 b) 30 c) 60 d) 120
10. Signature of an even permutation is _____.
a) -1 b) 1 c) 2 d) -2

Q-2 Answer any **TEN**

20

1. Prove that $(\mathbb{Z}_4, +)$ forms a group.
2. State and prove cancellation laws of a group.
3. Prove that G is abelian iff $G = Z(G)$.
4. Define Isomorphism.
5. Find all the cosets of $3Z$ in Z .
6. Prove that any infinite cyclic group is isomorphic to Z .
7. Define normal subgroup.
8. Define Simple group.
9. State second isomorphism theorem.
10. Explain signature of permutation with example.
11. Define Transposition.

[P.T.O.]

12. Express the permutation $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 2 & 1 & 4 & 3 & 6 & 5 \end{pmatrix}$ as the product of disjoint cycles.

Q-3

- (a) Let G be a set with associative binary operation. Assume that for all a, b in G , the equations $ax = b$ and $ya = b$ have unique solutions in G . Prove that G is a group. 5
- (b) Prove that the intersection of finite number of subgroups of a group G is again a subgroup of G . 5

OR

Q-3

- (a) Let H and K be finite subgroups of G such that HK is a subgroup of G . 5
Then prove that $|O(HK)| = \frac{|O(H)||O(K)|}{|O(H \cap K)|}$.
- (b) Let H be a finite subset of a group G such that $ab \in H$, whenever $a, b \in H$. 5
Prove that H is a subgroup of G .

Q-4

- (a) State and Prove Euler's theorem. 5
- (b) If H is subgroup of a group G then show that G is the union of all left cosets of H in G and any two left cosets of H in G are disjoint. 5

OR

Q-4

- (a) State and Prove Fermat's theorem. 5
- (b) State and Prove Lagrange's theorem for a finite group. 5

Q-5

- (a) State and Prove third theorem of isomorphism. 5
- (b) If G is a direct product (internal) of subgroups H and K , then G is isomorphic to the external direct product of H and K . Conversely, if $G = H \times K$ is the external direct product of H and K then H and K are isomorphic to subgroups H' and K' of G respectively, and G is a direct product (internal) of subgroups H' and K' . 5

OR

Q-5

- (a) Show that a subgroup H is normal in G if and only if $xH = Hx$, for each $x \in G$. 5
- (b) State and Prove first isomorphism theorem. 5

Q-6

- (a) Prove that S_n , the set of all permutations on n symbols is a group. 5
- (b) State and prove Sylow theorem I. 5

OR

Q-6

- (a) State and Prove Cauchy's theorem. 5
- (b) State and Prove Cayley's theorem. 5



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