



[68]

B.Sc. Sem : 5

Subject : Mathematics

Max. Marks : 70

Date: 04/10/2025

US05CMTH21 [Real Analysis]

Time : 10.00 to 1.00

Q.1 Choose the correct option for each of the following.

[10]

(1) is an complete ordered filed.

(a) Q (b) Z (c) R (d) None of these

(2) The smallest number of a set ,if exists is

(a) the supremum of the set (b) the infimum of the set (c) not unique (d) None of these

(3) The supremum of the $\left\{\frac{1}{n+1} / n \in N\right\}$ is

(a) 0 (b) 1 (c) 2 (d) None of these

(4) $\{1,2,3,4\}$ is Set.

(a) an open (b) a closed (c) open and closed (d) None of these

(5) The derived set of $A = (-1,1)$ is(a) $\emptyset$ (b) R (c) $(-1,1)$ (d) $[-1,1]$ (6) The interior of the set Z is(a) N (b) Z (c) $\emptyset$ (d) R

(7) The Range of a sequence is always

(a) empty (b) infinite (c) non- empty (d) None of these

(8) The range set of a sequence $\{(-1)^{n+1} / n \in N\}$ is(a) $\{-1,1\}$ (b) $\{1\}$ (c) $\{-1\}$ (d) None of these(9) A positive term series $\sum \frac{1}{n^2}$ is

(a) convergent (b) divergent (c) converges to 0 (d) None of these

(10) A series $\sum u_n$ is convergent then $\lim_{n \rightarrow \infty} u_n$

(a) = 0 (b) = 2 (c) = 1 (d) does not exists

Q.2 Attempt any TEN:

[20]

(1) Prove that the least upper bound of a set S is unique ,if it exist.(2) Prove that $\sqrt{3}$ is not a rational number.(3) Find the g.l.b and l.u.b of $\left\{\frac{1}{m} + \frac{1}{n} / m, n \in N\right\}$ if they exist.(4) Give an example to show that $(S \cap T)' \neq S' \cap T'$, where S and T are sets of real numbers.

(5) Define: A closure of a set.

(6) Define: A limit point of a set .

[11]

(P.T.O.)

(7) Define: A convergent sequence .

(8) Prove that every convergent sequence is bounded.

(9) Prove that $\lim_{n \rightarrow \infty} \frac{3 + \sqrt{2}n}{n} = \sqrt{2}$.

(10) State Cauchy's General principle of convergence.

(11) Prove that the series $\sum_{n=1}^{\infty} \frac{1}{n!}$ is convergent.

(12) Investigate the behaviour of the series $\frac{1}{2} + \frac{2}{3} + \frac{3}{4} + \frac{4}{5} + \dots$

Q.3 (a) State and Prove the Archimedean property of $\mathbb{R}$. [5]

(b) State and Prove triangular inequalities of real numbers. [5]

OR

Q.3 (a) Prove that $\mathbb{Q}$ is not an ordered complete field. [5]

(b) Prove that $\sqrt{6}$ is an irrational number. [5]

Q.4 (a) Prove that every infinite bounded set has a limit point. [5]

(b) Prove that the derived set of a set is a closed set. . [5]

OR

Q.4 (a) Prove that a subset F is closed in $\mathbb{R}$ if and only if $\mathbb{R} - F$ is open . [5]

(b) Give an example to show that the arbitrary intersection of open sets need not be open. [5]

Q.5 (a) State and Prove Cauchy's first theorem on limits. [5]

(b) If $\{a_n\}$ and $\{b_n\}$ are two sequences such that $\lim_{n \rightarrow \infty} a_n = a$ and $\lim_{n \rightarrow \infty} b_n = b$ then prove that $\lim_{n \rightarrow \infty} (a_n + b_n) = a + b$. [5]

OR

Q.5 (a) Prove that every bounded sequence with a unique limit point is convergent. [5]

(b) Prove that $\lim_{n \rightarrow \infty} \sqrt[n]{n} = 1$. [5]

Q.6 (a) State and Prove D'Alembert's Ratio Test for series.

(b) Prove that the positive term geometric series $1 + r + r^2 + \dots$ converges for $r < 1$ and diverges to ∞ for $r \geq 1$. [5]

OR

Q.6 (a) State and Prove comparison test of first type for a series. [5]

(b) Prove that the series $\frac{1.2}{3^2.4^2} + \frac{3.4}{5^2.6^2} + \frac{5.6}{7^2.8^2} + \dots$ converges. [5]

—X—
(2)