



A Charutar Vidya Mandal's Institution  
**V.P. & R.P.T.P. SCIENCE COLLEGE**  
(AUTONOMOUS) (Grant-in-Aid)  
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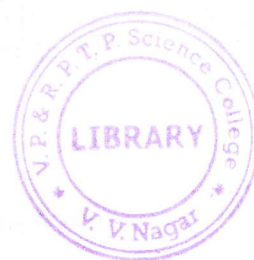
**Bachelor of Science (Semester-4) External Examination – 2026**  
**Subject Code:US04MAMTH02 Subject Name: Group Theory**

**Date: 25-03-2026, (Wednesday)**  
**Time: 01:45 PM to 04:15 PM (2.5 hrs)**

**Total marks: 50**

Q-1 Write the most appropriate answer from the following multiple-choice questions. [10]

- 1  $Z_n^*$  forms a group if  $n$  is ....  
(a) 6  
(b) 7  
(c) 4  
(d) none of these
- 2 Center of  $Z$  is .....  
(a)  $Z$   
(b)  $N$   
(c) 2  
(d)  $\{1, -1\}$
- 3 Cyclic group of order 5 has ..... Generators.  
(a) 6  
(b) 5  
(c) 4  
(d) 2
- 4 Every cyclic group of ..... order is simple group.  
(a) 4  
(b) even  
(c) prime  
(d) none of above
- 5 ..... is generator of group  $Z_n$ .  
(a)  $\bar{0}$   
(b)  $\bar{1}$   
(c)  $\bar{3}$   
(d) none of above
- 6 No. Of normal subgroups of  $Z_8$  are.....  
(a) 4  
(b) 2  
(c) 8  
(d) none of above
- 7 If  $H$  is any normal subgroup of  $G$  then .....  
(a)  $HX=XY$   
(b)  $HX=H$   
(c)  $HX=XH$   
(d)  $XH=YH$
- 8 The order of  $S_n/A_n$  is .....  
(a)  $n$   
(b)  $n!/2$   
(c)  $n!$   
(d) 2



- 9  $Z_5 \oplus Z_{11} \simeq \dots$   
 (a)  $Z_{15}$   
 (c)  $Z_{11}$   
 10 Signature of every transposition is .....  
 (a) 1  
 (b) -1
- (b)  $Z_{55}$   
 (d) none of these  
 (b) 2  
 (d) -2



Q-2 Prove that  $(G, \times)$  form a group. where  $G$  is the set of all  $2 \times 2$  non singular matrices. [10]  
 Is it abelian?

OR

Q-2 Let  $H$  &  $K$  be subgroups of group  $G$ . Then Prove that  $HK$  is a subgroup of  $G \Leftrightarrow HK=KH$ . [10]

- Q-3 (a) Prove that every subgroups of  $Z$  is of the form  $mZ$ , for some  $m \in Z$ . [05]  
 (b) Find all normal subgroups of  $(Z_{15}, +_{15})$ . [05]

OR

- Q-3 (a) State and prove Lagrange's Theorem. [04]  
 (b) Find order of every elements of  $(Z_6, +_6)$ . Also find all the generators of it. [06]

- Q-4 (a) State and prove first isomorphism theorem for group. [05]  
 (b) Let  $f: G \rightarrow G'$  be homomorphism of  $G$  onto  $G'$ . Then Prove that the  $\ker f$  is a normal subgroup of  $G$ . [05]

OR

- Q-4 (a) Let  $f: G \rightarrow G'$  be any group homomorphism then prove that  $f$  is one-one  $\Leftrightarrow \ker f = \{e\}$ . [05]  
 (b) P.T any finite cyclic group of order  $n$  is isomorphic to  $Z_n$ . [05]

- Q-5 (a) Prove that every permutation can be expressed as a product of disjoint cycles. [05]  
 (b) If  $\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 2 & 3 & 4 & 5 & 7 & 6 & 8 & 1 \end{pmatrix}$  find invers of it and express it as a product of disjoint cycles. [05]

OR

- Q-5 (a) State and prove Cayley's theorem. [05]  
 (b) Define Internal Direct Product and explain it with an example. [05]

\*\*\*\*\*All The Best\*\*\*\*\*