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A Charutar Vidya Mandal's Institution
V. P. & R. P. T. P. SCIENCE COLLEGE
(AUTONOMUS) (Grant-in-Aid)



Affiliated to Sardar Patel University, Vallabh Vidyanagar -388120

Bachelor of Science (Semester IV) External Examination-2026
US04MAMTH01: Mathematical Analysis-I

Date: 24-03-2026 (Tuesday)

Time: 1:45 pm to 4:15 pm (2.5 hrs)

Total marks: 50

NOTE: The symbols used in this paper have usual meaning unless specified.

Q. 1 Write the most appropriate answer.

[10]

- (1) A monotonically increasing sequence is
(a) always convergent. (b) always divergent.
(c) always bounded. (d) convergent only if bounded.
- (2) Which of the following statement/s implies convergence of the sequence $\{x_n\}$?
(a) $\forall \epsilon > 0, \exists N \in \mathbb{N} \ni |x_{n+1} - x_n| < \epsilon, \forall n \geq N$.
(b) For a convergent sequence $\{a_n\}$, $\exists N \in \mathbb{N} \ni |x_n - x_m| < a_n, \forall n, m \geq N$.
(c) For a sequence $\{a_n\}$, converges to 0, $\exists N \in \mathbb{N} \ni |x_n - x_m| < 0.2a_n, \forall n, m \geq N$.
(d) For a convergent sequence $\{y_n\}$, $x_n \leq y_n, \forall n \in \mathbb{N}$.
- (3) For $n \in \mathbb{N}$, let $x_n = (-1)^n \sin \frac{1}{n}$ and $y_n = \frac{1}{n^{1.00001}}$. Which of the following is TRUE?
(a) $\sum x_n$ is convergent and $\sum y_n$ does NOT converge.
(b) $\sum x_n$ does not converge and $\sum y_n$ converges.
(c) $\sum x_n$ converges and $\sum y_n$ converges.
(d) $\sum x_n$ does NOT converge and $\sum y_n$ does NOT converge.
- (4) If for a sequence $\{a_n\}$ of non-negative numbers $\frac{a_{n+1}}{a_n} \geq 1$ then
(a) the series $\sum a_n$ is divergent.
(b) the series $\sum a_n$ is convergent and $\sum a_n \geq 1$.
(c) the series $\sum a_n$ is convergent but cannot conclude about its sum.
(d) the given information is insufficient to conclude the behavior of the series $\sum a_n$.
- (5) The sum of the infinite series $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{\pi^{2n}}{4^{2n}(2n+1)!}$ is
(a) divergent. (b) convergent and its sum is 0.
(c) convergent and its sum is $\frac{\pi-2}{2\pi}$. (d) convergent and its sum is $\frac{2}{\pi}$.
- (6) If for a function $f : \mathbb{R} \rightarrow \mathbb{R}$, $\lim_{x \rightarrow a} f(x) = l$ then for any sequence $\{x_n\}$ converges to a ,
 $\lim_{n \rightarrow \infty} f(x_n)$

P.T.O.

(a) may not exist.

(b) exists but may not be equal to l .

(c) exists and is equal to l .

(d) exists and is equal to $f(a)$.

(7) For $A \subset \mathbb{R}$, let $f : A \rightarrow \mathbb{R}$ is continuous then

(a) f may not be bounded.

(b) f is bounded but do not attend its bounds.

(c) f is bounded and attend its bounds.

(d) f is bounded and $A \subset f(A)$.

(8) The Dirichlet function is

(a) nowhere continuous.

(b) continuous only at 0.

(c) continuous only on $\mathbb{Q}$.

(d) continuous only on $\mathbb{Q}^c$.

(9) The sequence $\left\{ \frac{2n+1}{3n+2} \right\}$

(a) is unbounded.

(b) is divergent and unbounded.

(c) is divergent and bounded.

(d) is convergent.

(10) Let $f(x) = x^3 - 5x + 3$ then

(a) f has no root in $(0, 1)$.

(b) f has a root in $(0, 1)$.

(c) f has no root in $\mathbb{R}$.

(d) cannot say anything about the root of f in $(0, 1)$.

Q. 2 (A) Prove that $\lim_{n \rightarrow \infty} \frac{1}{n^2+1} = 0$.

[05]

(B) Let $\{x_n\}$ and $\{y_n\}$ be sequence of real numbers that converges to x and y , respectively.

Then show that the sequence $\{x_n + y_n\}$ converge to $x + y$.

[05]

OR

Q. 2 (P) State and prove the Monotone Convergence theorem for increasing sequence.

[05]

(Q) State and prove Squeeze theorem.

[05]

Q. 3 (A) Show that if $\sum a_n$ converges then $a_n \rightarrow 0$ and that $a_n \rightarrow 0$ is not sufficient for convergence of $\sum a_n$.

[05]

(B) Check the convergence of the series: (i) $\sum_{n=1}^{\infty} \frac{1}{n^2+n}$ (ii) $\sum_{n=0}^{\infty} \frac{1}{n!}$.

[05]

OR

Q. 3 (P) State and prove the root test.

[05]

(Q) Using Cauchy's condensation test, prove that the p -series $\sum_{n=1}^{\infty} 1/n^p$ converges if and only if $p > 1$.

[05]

Q. 4 (A) Let $\{a_n\}$ and $\{b_n\}$ be sequences and define $A_n = \sum_{k=1}^n a_k$, $A_0 = 0$. Show that for $0 < p \leq q$, $\sum_{n=p}^q a_n b_n = \sum_{n=p}^q (A_n - A_{n-1}) b_n = \sum_{n=p}^q A_n b_n - \sum_{n=p-1}^{q-1} A_n b_{n+1}$.

[05]

(B) Show that every point of the open interval $[0, 1]$ is a cluster point of $(0, 1)$.

[05]

OR

Q. 4 (P) State a result to find the radius of convergence of the power series and find the radius of convergence of the series $\sum \frac{x^n}{n!}$.

[05]

(Q) State and prove the sequential criterion for limits of functions.

[05]

P.T.O.

Q. 5 (A) Prove that a continuous function on closed and bounded interval is bounded. [05]

(B) State and prove Bolzano's Intermediate Value theorem. [05]

OR

Q. 5 (P) Prove that every continuous function on closed and bounded interval attains its maximum. [05]

(Q) Prove that if $f : A \rightarrow \mathbb{R}$ is uniformly continuous on a subset A of $\mathbb{R}$ and if $\{x_n\}$ is a Cauchy sequence in A , then $\{f(x_n)\}$ is a Cauchy sequence in $\mathbb{R}$. [05]

All The Best

