



A Charutar Vidya Mandal's Institution

V. P. & R. P. T. P. SCIENCE COLLEGE

(AUTONOMUS) (Grant-in-Aid)

Affiliated to Sardar Patel University, Vallabh Vidyanagar -388120

**Bachelor of Science (Semester 3) External Examination-2025****US03MAMTH02: Multivariate Calculus and The Real Number system**

Date: 07-10-2025 (Tuesday)

Time: 2:00 pm to 4:30 pm

Total marks: 50

NOTE: The symbols used in this paper have usual meaning unless specified.**Q. 1 Write the most appropriate answer from the following multiple-choice questions. [10]**

- (1) The region $R : 1 \leq x \leq 2, 1 \leq y \leq x^2$ can be express as
 (a) $R : 0 \leq x \leq 1, x^2 \leq y \leq 1$. (b) $R : 0 \leq y \leq 2, 0 \leq x \leq \sqrt{y}$.
 (c) $R : 1 \leq y \leq 4, \sqrt{y} \leq x \leq 2$. (d) $R : 0 \leq y \leq 1, \sqrt{y} \leq x \leq 1$.
- (2) Area of the region R bounded by $y = x^2$ and $y = 2x - x^2$ is _____.
 (a) $\frac{1}{3}$ (b) $\frac{2}{3}$ (c) $\frac{5}{3}$ (d) $\frac{8}{3}$
- (3) The spherical coordinate equation for the cone $z = \sqrt{x^2 + y^2}$ is _____.
 (a) $\phi = \frac{\pi}{4}$ (b) $\phi = \frac{\pi}{3}$ (c) $\rho \sin \phi = 2$ (d) $\rho = 2 \cos \phi$
- (4) The value of $\int_{-1}^1 \int_0^{\sqrt{1-x^2}} (x^2 + y^2) dy dx =$ _____.
 (a) 0 (b) π (c) $\frac{\pi}{4}$ (d) $\frac{\pi}{8}$
- (5) If F is solenoid field then _____.
 (a) $\text{curl} F = 0$ (b) $\text{div} F = 0$ (c) $\text{grad} F = 0$ (d) none of these
- (6) The curve $r(t) = \cos t i + \sin t j + 2k, 0 \leq t \leq \pi$ is _____.
 (a) simple and closed curve. (b) simple but not closed curve.
 (c) neither simple nor closed curve. (d) closed but not simple curve.
- (7) If F is conservative defined on the domain D , then which of the following is **not** true.
 (a) $\oint_C F \cdot dr = 0$, around every closed curve in D . (b) F is a gradient field in D .
 (c) F has potential function f in D . (d) $\int_C F \cdot dr = 0$ for at least one curve C in D .
- (8) If $F = x^2 i + y^2 j + z^2 k$ defined on the hemisphere $x^2 + y^2 + z^2 = 1$ above xy - plane and $C : x^2 + y^2 = 1$ then $\oint_C F \cdot T ds =$ _____.
 (a) π (b) 2π (c) 0 (d) 1
- (9) Suppose E is bounded above subset of $\mathbb{Q}$. Then
 (a) $\sup E$ may not exist in $\mathbb{Q}$. (b) $\sup E$ exists in $\mathbb{Q}$ and E may not contain its supremum.
 (c) $\sup E$ exists in $\mathbb{Q}$ and E contains its supremum. (d) None of these.

- (10) Consider the statements: (i) For a set S , if there exists an injection from S into $\mathbb{N}$ then S is countable. (ii) There exist a set A with a surjection from A onto the set $P(A)$.

Which of the following is correct?

(a) both (i) and (ii) are true.

(b) (i) is true but not (ii).

(c) (ii) is true but not (i).

(d) both (i) and (ii) are not true.

Q. 2 (A) Find all the values of constant c such that $\int_0^1 \int_0^c (2x + y) dx dy = 3$. [02]

(B) Sketch the region of the integral $\int_0^2 \int_{x^2}^{2x} (4x + 2) dy dx$ and write an equivalent integral by changing the order of integration to find the volume of the solid formed by given function. [05]

(C) Define the double integral as a limit of sum for function $f(x, y)$ defined on the rectangular region R by taking partition of it and state the Fubini's theorem for rectangular region R . [03]

OR

Q. 2 (P) Find the area of the playing field described by $R : -2 \leq x \leq 2, -1 - \sqrt{4 - x^2} \leq y \leq 1 + \sqrt{4 - x^2}$ and sketch the region. [04]

(Q) Evaluate (i) $\int_0^1 \int_0^1 |x - y| dy dx$ and (ii) $\int_0^1 \int_0^1 (x - y) dy dx$. [06]

Q. 3 (A) Give the relation between Spherical, Cylindrical and Cartesian coordinates and find the spherical equation for (a) $x^2 + y^2 = z$, (b) $r(3 \cos \theta + 2 \sin \theta) + 6z = 0$, (c) $x^2 + y^2 = 16$. [05]

(B) Find the volume of the solid above xy -plane bounded by the elliptic paraboloid $z = x^2 + 4y^2$ and the cylinder $x^2 + 4y^2 = 4$ using triple integral. [05]

OR

Q. 3 (P) Define Jacobian of the coordinate transformation $x = g(u, v)$, $y = h(u, v)$ and evaluate $\int_{y=0}^4 \int_{x=\frac{y}{2}}^{1+\frac{y}{2}} \frac{2x-y}{2} dx dy$ by taking $u = \frac{2x-y}{2}$, $v = \frac{y}{2}$ over the region in uv -plane. [06]

(Q) If $\vec{r} = x \mathbf{i} + y \mathbf{j} + z \mathbf{k}$ then show that $\vec{F} = \frac{\vec{r}}{r^2}$ is irrotational. [04]

Q. 4 (A) Find the flux of $\vec{F} = (y^2 - 6x)\mathbf{i} + (2y - x^2)\mathbf{j}$ across the ellipse $C : x^2 + 4y^2 = 4$. [04]

(B) Verify Gauss's divergence theorem for $\vec{F}(x, y, z) = xy \mathbf{i} + yz \mathbf{j}$ and S is the cube bounded by the coordinate planes and the planes $x = 1$, $y = 1$, $z = 1$. [06]

OR

Q. 4 (P) State and prove Green's theorem in plane. [07]

(Q) Evaluate the surface integral $\iint_S f(x, y, z) d\sigma$ where $f(x, y, z) = x$ and S is the portion of the plane $x + y + z = 1$ in the first octant. [03]

Q. 5 (A) Suppose S is an ordered set with least upper bound property, $B \subset S$ is non empty bounded below. Let L be the set of all lower bounds of B . Then $\alpha = \sup L$ exists in S , and $\alpha = \inf B$. [05]

(B) Prove that $[0, 1]$ is uncountable. [05]

OR

Q. 5 (P) Prove that if $x \in \mathbb{R}$, $y \in \mathbb{R}$, and $x < y$, then there exists a $p \in \mathbb{Q}$ such that $x < p < y$. [05]

(Q) Prove that if $I_n = [a_n, b_n]$, $n \in \mathbb{N}$, is a nested sequence of closed bounded intervals, then there exists a number $\xi \in \mathbb{R}$ such that $\xi \in I_n$ for all $n \in \mathbb{N}$. [05]

Best of Luck

