



A Charutar Vidya Mandal's Institution
V.P. & R.P.T.P. SCIENCE COLLEGE
 (AUTONOMOUS) (Grant-in-Aid)
 Affiliated to Sardar Patel University
 VALLABH VIDYANAGAR-388120



Bachelor of Science (Semester- III) External Examination – 2025
US03MAMTH01: Linear Algebra

Date: 06-10-2025, (Monday)

Time: 02:00 PM to 04:30 PM (2.5 hrs)

Total marks: 50

NOTE: All notations are standard.

Q-1 Write the most appropriate answer from the following multiple-choice questions.

- (1) The set $\{1, x, x^2, \dots\}$ in the vector space of real polynomials is _____.
 (a) finite basis (b) infinite basis (c) dependent (d) not a basis
- (2) The set $\{(1,1,1), (2,3,4), (0,1,1)\}$ in R^3 is _____.
 (a) linear independent (b) linear dependent (c) basis (d) none
- (3) The kernel of the map $T(x+z, y+z)$ is spanned by _____.
 (a) $(1,0,-1)$ (b) $(0,1,-1)$ (c) $\{(1,0,-1), (0,1,-1)\}$ (d) none
- (4) The composition of two linear transformations is _____.
 (a) always linear (b) nonlinear (c) sometimes linear (d) none
- (5) The Wronskian of $\{e^x, e^{-x}\}$ is _____.
 (a) 0 (b) -2 (c) 2 (d) 1
- (6) A reflection matrix in 2 dimension has determinant _____.
 (a) 1 (b) -1 (c) 0 (d) 2
- (7) A real symmetric matrix is always _____.
 (a) diagonalizable (b) non-diagonalizable (c) skew symmetric (d) none
- (8) The quadratic form $x^2 + 2xy + y^2$ corresponds to matrix _____.
 (a) $\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$ (c) $\begin{bmatrix} 1 & 1 \\ 2 & 1 \end{bmatrix}$ (d) $\begin{bmatrix} 1 & 2 \\ 1 & 1 \end{bmatrix}$
- (9) An orthogonal matrix Q satisfies _____.
 (a) $Q^T Q = I$ (b) $Q Q^T = I$ (c) columns form orthonormal set (d) all of the above
- (10) The Abel's theorem applies to _____.
 (a) algebraic equations (b) linear differential equations
 (c) non-linear equations (d) none of these

- Q-2 (a)** Check whether the subset $S = \{(1,2,3), (3,1,0), (-2,1,3)\}$ forms a basis for V_3 or not? In case, S is not a basis for V_3 , determine a basis for $[S]$. **[05]**
- (b)** Let V be a vector space. For every $u \in V$ and scalar α , prove that **[05]**
- (i) $\alpha 0 = 0$ (ii) $0u = 0$ (iii) $(-1)u = -u$.

OR



Q-2 (a) Show that a nonempty subset S of a vector space V is a subspace of V if and only if the following conditions are satisfied: (i) if $u, v \in S$, then $u + v \in S$. [05]
(ii) If $u \in S$ and a scalar α , then $\alpha u \in S$.

(b) Prove that if U and W are two subspaces of a finite – dimensional vector space V , then $\dim(U + W) = \dim(U) + \dim(W) - \dim(U \cap W)$. [05]

Q-3 (a) Prove that a linear transformation T is completely determined by its values on the elements of a basis. [05]

(b) Show that the linear map $T: V_3 \rightarrow V_3$ defined by $T(e_1) = e_1 - e_2, T(e_2) = 2e_2 + e_3, T(e_3) = e_1 + e_2 + e_3$ is neither one-one nor onto. [05]

OR

Q-3 (a) Prove that a transformation $T: \mathbb{R}^3 \rightarrow \mathbb{R}^3, T(x, y, z) = (x + y + z, y + z, z)$ is non-singular and also find T^{-1} . [05]

(b) State and prove Rank-Nullity theorem for linear transformation. [05]

Q-4 (a) Prove that the dimension of the vector space M_{mn} is mn . [05]

(b) Let a linear map $T: V_3 \rightarrow V_3$ be defined by [05]

$$T(x_1, x_2, x_3) = \left(x_1 - x_2 + x_3, 2x_1 + 3x_2 - \frac{1}{3}x_3, x_1 + x_2 - 2x_3\right).$$

Find the matrix associated with linear map related to the basis

$$B_1 = \{(1,0,0), (0,1,0), (0,0,1)\} \text{ and } B_2 = \{(1,1,0), (1,2,3), (-1,0,1)\}.$$

OR

Q-4 (a) Prove that the range of an $m \times n$ matrix is the span of its column vector. [05]

(b) If y_1, y_2 are solutions of $a_0 y'' + a_1 y' + a_2 y = 0$ where a_0, a_1 and a_2 belongs to $\mathcal{C}(I)$, and a_0 is nonzero here on I , then prove that [05]

$$W[y_1(x), y_2(x)] = C e^{-\int \frac{a_1(x)}{a_0(x)} dx}.$$

Q-5 (a) Let V be an inner product space. Prove that (i) $\|\alpha u\| = |\alpha| \|u\|$ [05]

(ii) $\|u + v\| \leq \|u\| + \|v\|$ for arbitrary vectors $u, v \in V$ and scalar α .

(b) Find an orthonormal basis of $f_3[-1,1]$ starting from the basis $\{1, x, x^2, x^3\}$. [05]

Use the inner product defined by $f \cdot g = \int_{-1}^1 f(t)g(t)dt$.

OR

Q-5 (a) Prove that any orthogonal set of non-zero vectors in an inner product space is linearly independent. [05]

(b) Reduce the quadratic form $Q(x, y) = 3x^2 + 4xy + 3y^2$ to its canonical form by diagonalization of the associated symmetric matrix. [05]



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