V.P.& R.P.T.P.Science College.Vallabh Vidyanagar.
Internal Test B.Sc. Semester-VI
US06CMTH04 (Abstract Algebra-2)
Thursday , 15^{th} March 2018 11.00 a.m. to 12.30 p.m. Maximum Marks :25

Que.1 Fill in the blanks. 3

(1) In Ring of real quaternion , $(1-2i-3j-2k)^{-1} =$ ....

(a) $\frac{1-2i-3j-2k}{18}$ (b) $\frac{1+2i+3j+2k}{18}$ (c) $\frac{-1+2i+3j+2k}{18}$ (d) $\frac{1-2i-3j-2k}{6}$

(2) .... is maximal ideal of field .

(a) 0 (b) $\{1\}$ (c) $\{0\}$ (d) none of these

(3) $1+2i$ and .... are associates in the ring of Gaussian integer .

(a) $2+i$ (b) $-2+i$ (c) i (d) $2+i$

Que.2 Answer the following (Any Two) 4

(1) Prove that Z_p is a field , where p is prime.

(2) Define : Quotient field of ID , Proper ideal , Prime ideal , Maximal ideal .

(3) Let$R = \{a+b\sqrt{-5}/a, b \in Z\}$.Show that $1+2\sqrt{-5}$ and 3 are relatively prime.

Que.3 (a) Let R be the set of all subsets of a set X . Define $+ \; and \cdot$ in R by $A+B=(A-B)\cup(B-A)$; $A \cdot B = A \cap B$ .Prove that the set R forms a ring. 3

(b) Prove that the characteristic of a field is either 0 or a prime. 3

OR

Que.3 (a) State and prove Cayley's theorem for rings. 3

(b) Under which condition every integral domain is a field. Verify it. 3

Que.4 (a) Prove that every field is a simple ring . Under which condition converse holds.Verify it. 6

OR

Que.4 (a) State and prove First isomorphism theorem for ring. 3

(b) Prove that an ideal M in Z is a maximal ideal iff $M = pZ$, where p is a prime. 3

Que.5 (a) Prove that every principal ideal domain is factorization domain. 3

(b) Show that the ring of Gaussian integers is Euclidean domain. 3

OR

Que.5 (a) Prove that every prime element is irreducible in integral domain with unit element 1 . Does the converse hold ? Verify it. 6

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