V.P.& R.P.T.P.Science College.Vallabh Vidyanagar.
Internal Test
B.Sc. Semester VI
US06CMTH04 (Abstract Algebra -2)
Thursday , 9^{th} March 2017
11.00 a.m. to 12.30 p.m.

Maximum Marks: 25

Que.1 Fill in the blanks. 3

(1) ..... is regular element of $\mathbb{Z}_9$.
(a) 3 (b) 4 (c) 6 (d) none of these

(2) If I is ideal in ring R then unit element of R/I is .....
(a) 0 (b) 1 (c) R (d) $1+I$

(3) Every ..... has unit element.
(a) integral domain (b) ring (c) Euclidean domain (d) commutative ring

Que.2 Answer the following (Any Two) 4

(1) Prove that the mapping $f : C \to C$ defined by $f(z) = \bar{z}$ is a ring isomorphism .

(2) Let $f : R \to R'$ be ring homomorphism ,then prove that Kerf is an ideal in R.

(3) Prove that $1 + 2i$ and $2 - i$ are associates in $Z + iZ$.

Que.3 (a) Prove that the set $R = \left\{ \begin{bmatrix} a & -b \\ b & a \end{bmatrix} / \ a, b, \in \mathbb{R} \right\}$ forms a ring. 3

(b) State and prove Cayley's theorem for rings. 3

OR

Que.3 (a) Prove that every finite integral domain is a field. 3

(b) Prove that the only isomorphism of $\mathbb{C}$ onto $\mathbb{C}$ which maps reals to reals is the identity map $I_{\mathbb{C}}$ or the conjugation map. 3

Que.4 (a) Let $F = \{\overline{(a,b)}/a, b \in R, b \neq 0\}$ be the set of all equivalence class of elements of $\{(a,b)/a, b \in R, b \neq 0\}$.Define $+$ *and* $\cdot$ in F by
$\overline{(a,b)} + \overline{(c,d)} = \overline{(ad+bc, bd)}$, $\overline{(a,b)} \cdot \overline{(c,d)} = \overline{(ac, bd)}$.
Prove that F is a field.Is F an integral domain? 6

OR

Que.4 (a) State and prove First isomorphism theorem for ring. 4

(b) Prove that an ideal P in commutative ring R is a prime ideal, if R/P is an integral domain. 2

Que.5 (a) Prove that every principal ideal domain is factorization domain. 4

(b) Let$R = \{a + b\sqrt{-5}/a, b \in Z\}$.Show that $2(1 + \sqrt{-5})$ and 6 have no gcd in R. 2

OR

Que.5 (a) Prove that every prime element is irreducible in integral domain with unit element 1 . Does the converse hold ? Verify it. 6

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