V.P. & R.P.T.P. Science College, V.V.Nagar

Internal Test: 2016-17

Subject : Mathematics   US06CMTH03   Max. Marks : 25

Topology

Date: 08/03/2017   Timing: 11:00 am - 12:30 pm

Q: 1. Answer the following by choosing correct answers from given choices. 3

[1] The discrete topology on a non-empty set X is ______ its indiscrete topology
[A] coarser than [B] finer than [C] not comparable with [D] none

[2] If A is a closed set in a topological space then
[A] $A \subset A'$ [B] $A^- \neq A$ [C] $A = A'$ [D] $A' \subset A$

[3] Every non-empty and bounded below subset of R posseses
[A] the g.l.b. in R [B] the l.u.b. in R
[C] g.l.b. and l.u.b. in R [D] none

Q: 2. Answer any TWO of the following. 4

[1] For a set $X = \{a, b, c, d\}$ give any two closed subsets of X relative to the topology $\{\emptyset, X, \{a\}, \{a, b\}\}$

[2] Find the sets of cluster points of $(1, 2)$ in usual topology and discrete topology of $\mathbb{R}$

[3] Let $f : [0, 1] \to R$ be continuous on $[0, 1]$ and be onto R also. Is $f([0, 1])$ connected?

Q: 3 [A] Let $\mathcal{G}$ be a family of subsets of $\mathbb{R}$ as described below
(i) $\emptyset \in \mathcal{G}$
(ii) If $G \in \mathbb{R}$ and $G \neq \emptyset$ then $G \in \mathcal{G}$ if for each $p \in G$ there is a set $H = \{x \in \mathbb{R}/a \leqslant x < b\}$ for some $a < b$ such that $p \in H \subset G$.
Prove that $\mathcal{G}$ is an unusual nontrivial topology of $\mathbb{R}$ 3

[B] Define Comparable Topologies and if $X = \{a, b, c\}$ then find three topologies $\mathcal{T}_1$, $\mathcal{T}_2$ and $\mathcal{T}_3$ for X such that $\mathcal{T}_1 \subsetneqq \mathcal{T}_2 \subsetneqq \mathcal{T}_3$ 3

OR

Q: 3 [A] Let $(X, \mathcal{T})$ be a topological space and let A be a subset of X. Prove that A is $\mathcal{T}$-open set iff A contains a $\mathcal{T}$-neighbourhood of each of its points 3

[B] Let $(X, \mathcal{T})$ be a topological space. If $\{F_\alpha / \alpha \in \Lambda\}$ is any collection of $\mathcal{T}$-closed subsets of X then prove that $\bigcap\{F_\alpha / \alpha \in \Lambda\}$ is a $\mathcal{T}$-closed set 3

Q: 4 [A] Let $(X, \mathcal{T})$ be a topological space and let A be a subset of X and A' be the set of all cluster points of A. Prove that A is $\mathcal{T}$-closed iff $A' \subset A$ 3

[B] Let $(X, \mathcal{T})$ be a topological space and let A be a subset of X. Then prove that $A^- = A \cup A'$. 3

OR

Q: 4. If $(X, \mathcal{T})$ and (Y, Ψ) are topological spaces and f is a mapping from X into Y then prove that the following statements are equivalent

(a) The mapping f is continuous

(b) The inverse image of f of every Ψ-closed set is $\mathcal{T}$-closed set

(c) If $x \in X$ then inverse image of every Ψ-neighbourhood of $f(x)$ is a $\mathcal{T}$-neighbourhood of x

(d) If $x \in X$ and N is a Ψ-neighbourhood of (x), then there is a $\mathcal{T}$-neighbourhood M of x such that $f(M) \subset N$

(e) If $A \subset X$, then $f(A^-) \subset f(A)^-$ 6

Q: 5 [A] Prove that if a space (X, T) has a nonempty proper subset A that is both $\mathcal{T}$-open and $\mathcal{T}$-closed, then $(X, \mathcal{T})$ is disconnected. 3

[B] If $(Y, \mathcal{T}_Y)$ is a compact subspace of a Hausdorff space $(X, \mathcal{T})$, then prove that Y is $\mathcal{T}$ closed. 3

OR

Q: 5. Prove that the space $(R, \mathcal{U})$ is connected. 6