

V.P.& R.P.T.P.Science College.Vallabh Vidyanagar.
 Internal Test
 B.Sc. Semester VI
 US06CMTH02 (Complex Analysis)
 Tuesday , 7th March 2017
 11.00 a.m. to 12.30 p.m.

Maximum Marks: 25

Que.1 Fill in the blanks.

- (1) Cartesian form of $f(z) = z^2$ is $f(z) = \dots\dots\dots$ 3
 (a) $x^2 + y^2 + i2xy$ (b) $x^2 - y^2 + i2xy$ (c) $x^2 + y^2 - i2xy$ (d) $x^2 - y^2 - i2xy$
- (2) $f(z) = \frac{z^3 + 4}{(z^2 - 3)(z^2 + 1)}$ is analytic in $\dots\dots\dots$
 (a) $\{\pm\sqrt{3}, \pm i\}$ (b) $\mathbb{C} - \{\sqrt{3}, i\}$ (c) $\mathbb{C} - \{\sqrt{3}, \pm i\}$ (d) none of these
- (3) e^z is periodic function with period $\dots\dots\dots$, $n \in \mathbb{Z}$.
 (a) $n\pi i$ (b) $2n\pi$ (c) $2n\pi i$ (d) $(2n + 1)\pi i$

Que.2 Answer the following (Any Two)

- (1) If $\lim_{z \rightarrow z_0} f(z) = w_0$, $\lim_{z \rightarrow z_0} g(z) = w_1$. Then prove that $\lim_{z \rightarrow z_0} [f(z)g(z)] = w_0w_1$.
- (2) Prove that $f'(z)$ does not exist at any point for $f(z) = e^{\bar{z}}$.
- (3) Solve the equation $e^{2z-1} = 1$.



Que.3 (a) By using definition of limit prove that $\lim_{z \rightarrow 2i} (2x + iy^2) = 4i$.

(b) If $f(z) = \frac{x^3y(y - ix)}{z(x^6 + y^2)}$, $z \neq 0$, $f(0) = 0$. Is $f(z)$ continuous at 0?

OR

Que.3 (a) Prove that $f(z) = |z|^2$ is differentiable only at $z = 0$.

(b) By using definition, prove that $\frac{d}{dz}(z^n) = nz^{n-1}$ for all $n \in \mathbb{Z}$.

Que.4 (a) State and prove sufficient conditions for differentiability of $f(z)$.

OR

Que.4 (a) Find harmonic conjugate of $u(x, y) = 2x - x^3 + 3xy^2$.

(b) Prove that $f'(z)$ and $f''(z)$ exist everywhere and find $f''(z)$ for $f(z) = iz + 2$

Que.5 (a) Prove that $\cosh^{-1}z = \log[z + \sqrt{z^2 - 1}]$.

(b) Prove that $\overline{\exp(iz)} = \exp(i\bar{z})$ iff $z = n\pi$, $n \in \mathbb{Z}$.

OR

Que.5 (a) Find all values of $\sin^{-1}(-i)$.

(b) Prove that $|\sinh z|^2 = \sinh^2x + \sin^2y = \cosh^2x - \cos^2y$.

