V.P. & R.P.T.P. Science College, V.V.Nagar

Internal Test: 2016-17

Subject : Mathematics | US06CMTH01 | Max. Marks : 25

Real Analysis - III

Date: 06/03/2017 | Timing: 11:00 am - 12:30 pm

Q: 1. Answer the following by choosing correct answers from given choices. **3**

[1] In usual notations, the *Schlömilch-Röche* form of remainder in Taylor's theorem is

[A] $\dfrac{h^{n-1}(1-\theta)^{n-p}}{p(n-1)!}f^{(n)}(a+\theta h)$ [B] $\dfrac{h^{n}(1-\theta)^{n-p}}{p(n-1)!}f^{(n)}(a+\theta h)$

[C] $\dfrac{h^{n}(1-\theta)^{n-p}}{p(n-1)!}f^{(n-1)}(a+\theta h)$ [D] $\dfrac{h^{n}(1-\theta)^{n}}{p(n-1)!}f^{(n)}(a+\theta h)$

[2] If a number c is a stationary point of derivable function f then

[A] $f(c)=0$ [B] $f'(c)=0$ [C] $f''(c)=0$ [D] none

[3] The norm of the partition $\{0,1,4,5,6,8,10\}$ of $[0,10]$ is

[A] 0 [B] 1 [C] 2 [D] 3

Q: 2. Answer any TWO of the following. **4**

[1] Explain the algebraic meaning of Rolle's theorem

[2] Show that $f(x)=x^3$ has no extreme value at 0

[3] Can two partitions of $[a,b]$ be disjoint sets? Justify.

Q: 3 [A] State and prove Lagrange's Mean Value theorem **3**

[B] Examine the validity of the hypothesis and the conclusion of Lagrange's Mean Value theorem for the function $f(x)=x(x-1)(x-2)$ on $[0,\frac{1}{2}]$ **3**

OR

Q: 3 [A] State and prove Taylor's theorem. **3**

[B] A twice differentiable function f is such that $f(a)=f(b)=0$ and $f(c)>0$ for $a<c<b$. Prove that there is at least one value ξ between a and b for which $f''(\xi)<0$. **3**

Q: 4 [A] Prove that if $f(c)$ is an extreme value of a function then $f'(c)$, if exists, is zero. **3**

[B] Show that $x^5-5x^4+5x^3-1$ has maxima at $x=1$, minima at $x=3$ and neither at 0 **3**

OR

Q: 4 [A] If c is an interior point of the domain $[a, b]$ of a function f and is such that
(i) $f'(c) = f''(c) = f'''(c) = ... = f^{(n-1)}(c) = 0$ and
(ii) $f^{(n)}$ exists and is zero
then show that for n odd, $f(c)$ is not an extreme value, while for n even $f(c)$ is maximum or minimum according as $f^{(n)}$ is negative or positive. 3

[B] Examine the function $\sin x + \cos x$ for extreme values 3

Q: 5. State and prove Darboux's Theorem. 6

OR

Q: 5. Prove that a necessary and sufficient condition for the integrability of a bounded function f is that for every $\epsilon > 0$ there exists a partition P of $[a, b]$ such that

$$U(P, f) - L(P, f) < \epsilon$$

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