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V.P.& R.P.T.P.Science College, Vallabh Vidyanagar.
B.Sc.(Semester - V) Internal Test
US05CMTH05 (Number Theory)

Date. 5/10/2016 ; Wednesday 11.00 a.m. to 12.30 p.m. Maximum Marks: 25

Que.1 Fill in the blanks. 3

(1) $(525, 231) =$ ....

(a) 10 (b) 31 (c) 21 (d) 7

(2) T(60) = ...

(a) 60 (b) 12 (c) 18 (d) 61

(3) 765432 is divided by ....

(a) 5 (b) 3 (c) 11 (d) 13

Que.2 Answer the following (Any Two) 4

(1) Prove that $[a, b, c] = [[a, b], c]$.

(2) Find highest power of 4 in 50! .

(3) Prove that the indeterminate equation $ax + by = c$ has solution iff d/c , where $(a, b) = d$.

Que.3 (a) Let g be a positive integer greater than 1 then prove that every positive integer a can can be written uniquely in the form $a = c_n g^n + c_{n-1} g^{n-1} + ... + c_1 g + c_0$, where $n \geq 0$, $c_i \in \mathbb{Z}$, $0 \leq c_i < g$, $c_n \neq 0$. 4

(b) Prove that $(a, b) = (a + kb, a)$, for $k \in Z$. 2

OR

Que.3 (a) State and prove Fundamental theorem of divisibility . 4

(b) If $(a, b) = 1$ then prove that $(ac, b) = (c, b)$. 2

Que.4 (a) Prove that $S(a) < a\sqrt{a}$, $\forall$ $a > 2$. 4

(b) Prove that $[x] + [y] \leq [x + y] \leq [x] + [y] + 1$. 2

OR

Que.4 (a) Prove that the necessary and sufficient condition that a positive integer a can be even perfect number is $a = 2^n(2^{n+1} - 1)$, $(n > 1)$ and $2^{n+1} - 1$ is prime . 4

(b) Prove that$u_{n+1}^2 = u_n^2 + 3u_{n-1}^2 + 2[u_{n-2}^2 + u_{n-3}^2 + \cdots + u_1]$. 2

Que.5 Prove that a general integer solution of $x^2 + y^2 + z^2 = w^2$, $(x, y, z, w) = 1$ is given by $x = (a^2 - b^2 + c^2 - d^2)$, $y = 2ab - 2cd$, $z = 2ad + 2bc$, $w = a^2 + b^2 + c^2 + d^2$. 6

OR

Que.5 (a) Solve the equation $19x + 20y = 1909$. 3

(b) Prove that every number containing more than two digits can be divided by 4 iff the number formed by last two digits can be divided by 4 . 3

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