V.P. & R.P.T.P. Science College, V.V.Nagar

Internal Test: 2016-17

Subject : Mathematics   US05CMTH03   Max. Marks : 25

Metric Spaces

Date: 03/10/2016   Timing: 11.00 am - 12.30 pm

Q: 1. Answer the following by choosing correct answers from given choices. **3**

[**1**] The set of cluster points of the null set, ϕ in $\mathbb{R}^1$ is
[A] R [B] Q [C] N [D] ϕ

[**2**] In a metric space (M, ρ), its subsets M and ϕ are
[A] both open but not closed   [B] closed but not open
[C] open as well as closed   [D] neither open nor closed

[**3**] If a subset A of a metric space M is totally bounded then it is
[A] complete   [B] unbounded   [C] bounded   [D] connected

Q: 2. Answer any TWO of the following. **4**

[**1**] Let X be a nonempty set and then find and $d : X \times X \longrightarrow R$ be defined by
$d(x, y) = \begin{cases} 0 & ; \text{if } x = y \\ 1 & ; \text{if } x \neq y \end{cases}$ then show that d is a metric on X.

[**2**] For the discrete metric $\mathbb{R}_d$, find (1) $B[a; 2]$ (2) $B[a; 1/2]$

[**3**] Prove or disprove that the empty set ϕ and singletone set are assumed to be connected.

Q: 3. Define limit of a function. Also prove that
$\lim\limits_{x \longrightarrow a} [f(x).g(x)] = \lim\limits_{x \rightarrow a} f(x). \lim\limits_{x \rightarrow a} g(x)$ **6**

OR

Q: 3 [A] Let $d : [0, \frac{\pi}{2}] \times [0, \frac{\pi}{2}] \longrightarrow \mathbb{R}$ be defined by $d(x, y) = \sin |x - y|$. Show that d is a metric on $[0, \frac{\pi}{2}]$. **3**

[**B**] If ρ and σ are metrics for M and if there exists $k > 1$ such that

$$\frac{1}{k}\sigma(x, y) \leqslant \rho(x, y) \leqslant k\sigma(x, y), \quad \forall x, y \in M$$

then prove that ρ and σ are eqivalent. **3**

Q: 4 [A] If E is any subset of the metric space M, Then show that $\overline{E}$ is closed. **3**

[**B**] If G_1 and G_2 are open subsets of the metric space M, then $G_1 \cap G_2$ is also open in M. **3**

OR

Q: 4 [A] Prove that $[0, 1]$ and $[a, b]$ are homeomorphic. **3**

[B] Let f be a continuous function from a metric space M_1 into a metric space M_2. If M_1 is connected, then prove that the range of f is also connected. **3**

Q: 5 [A] If the subset A of the metric space (M, ρ) is totally bounded, then prove that A is bounded. **3**

[B] If (M, ρ) is a complete metric space and A is a closed subset of M, then prove that (A, ρ) is also complete. **3**

OR

Q: 5. State and prove generalized nested interval theorem. **6**