V.P. & R.P.T.P. Science College, V.V.Nagar

Internal Test: 2016-17

Subject : Mathematics   US05CMTH02   Max. Marks : 25

Real Analysis-II

Date: 30/09/2016   Timing: 11.00 am - 12.30 pm

Q: 1. Answer the following by choosing correct answers from given choices.   **3**

[1] The sequence $\{2^n\}$
[A] is convergent [B] diverges to ∞ [C] diverges to $-\infty$ [D] oscillates finitely

[2] A positive term series $\sum\limits_{n=1}^{\infty} \frac{1}{n^p}$ is convergent if and only if
[A] $p < 1$ [B] $p > 1$ [C] $p \leqslant 1$ [D] $p \geqslant 1$

[3] For $f(x,y) = x^3 - 3xe^y$ the value of $f_x(1,0)$ is
[A] 0   [B] 1   [C] 2   [D] 3

Q: 2. Answer any TWO of the following.   **4**

[1] Prove that every convergent sequence is bounded.

[2] If $\sum\limits_{n=1}^{\infty} u_n = u$ and $\sum\limits_{n=1}^{\infty} v_n = v$ then prove that $\sum\limits_{n=1}^{\infty} (u_n - v_n) = u - v$

[3] Evaluate : $\lim\limits_{(x,y)\to(3,1)} \frac{\tan^{-1}(xy-3)}{\tan^{-1}(2xy-6)}$

Q: 3 [A] State and prove the Bolzano-Weierstarss theorem for sequence   **3**

[B] Prove that a sequence cannot converge to more than one limits.   **3**

OR

Q: 3. State and prove the Cauchy's criteria for the convergence of a sequence   **6**

Q: 4 [A] Show that a positive term series converges *iff* the sequence of its partial sums is bounded above.   **3**

[B] Test the series $\sum \frac{n^2-1}{n^2+1} x^n$ for convergence.   **3**

OR

Q: 4. State and prove the comparision test of first type in limit form.   **6**

Q: 5 [A] Show that $\lim\limits_{(x,y)\to(0,0)} xy \frac{x^2 - y^2}{x^2 + y^2} = 0$ **3**

[B] For the following function show that the repeated limits exist but the double limit does not when $(x,y) \to (0,0)$
$f(x,y) = \frac{x-y}{x+y}$ **3**

OR

Q: 5. If V is a function of two variables x and y and $x = r\cos\theta$, $y = r\sin\theta$ then prove that
$$\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} = \frac{\partial^2 V}{\partial r^2} + \frac{1}{r^2}\frac{\partial^2 V}{\partial \theta^2} + \frac{1}{r}\frac{\partial V}{\partial r}$$

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