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V.P. & R.P.T.P. Science College, V.V.Nagar

Internal Test: 2016-17

Subject : Mathematics US05CMTH01 Max. Marks : 25

Real Analysis-I

Date: 29/09/2016 Timing: 11.00 am - 12.30 pm

Q: 1. Answer the following by choosing correct answers from given choices. **3**

[1] The field which does not have the least upper bound property is
[A] N [B] Z [C] Q [D] R

[2] If S_1, S_2, S_3 and S_4 are open sets then $S_1 \cap S_2 \cap S_3 \cap S_4$ is
[A] closed [B] open [C] Open as well as closed [D] none

[3] If a function $f(x)$ has a discontinuity of first type at $x = 2$ then $\lim_{x\to 2-} f(x)$ and $\lim_{x\to 2+} f(x)$ both
[A] do not exist [B] exist and they are equal
[C] exist but they are not equal [D] cannot exist togather

Q: 2. Answer any TWO of the following. **4**

[1] Find the Greatest and the smallest members of $\left\{\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \dots\right\}$ if they exist.

[2] Determine whether the interior of the set $[2,8] \cup (9,10) \cap N$ is open or not.

[3] If $[x]$ denotes the largest integer less than or equal to x, then discuss the continuity at $x = 3$ for the function $f(x) = x - [x],\ \forall x \geqslant 0$,

Q: 3. State the Least Upper Bound property of R and prove that the field of rational numbers is not order complete. **6**

OR

Q: 3 [A] In usual notations prove that $E(x) = e^x,\ x \in R$. **3**

[B] State and prove the Archimedean property of R. **3**

Q: 4. Define Derived Set. Also prove that derived set S' of a bounded infinite set S has the smallest and the greatest members. **6**

OR

Q: 4 [A] Show that the interior of a set contains every open subset of a set. **3**

[B] If S and T are sets of real numbers then prove the following
(i) $S \subset T \Rightarrow S' \subset T'$ (ii) $(S \cup T)' = S' \cup T'$ **3**

Q: 5 [A] Examine the following function for continuity at $x = 0$

$$f(x) = \begin{cases} \dfrac{xe^{\frac{1}{x}}}{1+e^{\frac{1}{x}}} & \text{when} \quad x \neq 0 \\ 0, & \text{when} \quad x = 0 \end{cases}$$

3

[B] Prove that limit of a function is unique, if it exists. 3

OR

Q: 5. Show that a function $f : [a, b] \to \Re$ is continuous at point c of [a,b] iff

$$\lim_{n\to\infty} c_n = c \Longrightarrow \lim_{n\to\infty} f(c_n) = f(c)$$

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