V.P. & R.P.T.P. Science College, V.V.Nagar

Internal Test : 2013-14 T.Y.B.Sc. : Semester - 6 (CBCS)

Subject : Mathematics

US06CMTH01 Real Analysis - 3

Max. Marks : 30

Date: 10/03/2014

Timing: 3.30 pm - 5.00pm

Instructions : (1) This question paper contains FOUR QUESTIONS

(2) The figures to the right side indicate full marks of the corresponding question/s

(3) The symbols used in the paper have their usual meaning, unless specified

Q: 1. Answer any THREE of the following.

- [1] Explain the geometric meaning of Lagrange's Mean Value theorem
- [2] State Rolle's theorem
- $\begin{bmatrix} 3 \end{bmatrix}$ Can two partitions of [a, b] be disjoint sets? Justify.
- [4] Find the mesh of the partition $\{2, 3, 5, 7, 10, 11, 13\}$ of [2, 13]
- $\begin{bmatrix} 5 \end{bmatrix}$ Is f(x) = [x] an integrable function over [0, 5]? Justify.
- [6] If a function f has five points of discontinuity in [1,4] then can it be integrable over [1,4]? Justify.
- **Q: 2** [A] A twice differentiable function f is such that f(a) = f(b) = 0 and f(c) > 0 for a < c < b. Prove that there is al least one value ξ between a and b for which $f''(\xi) < 0$.
 - **[B]** Show that $\frac{\sin \alpha \sin \beta}{\cos \beta \cos \alpha} = \cot \theta$, for some θ where $0 < \alpha < \theta < \beta < \frac{\pi}{2}$

OR

Q: 2 [A] State and prove Maclaurin's theorem.

- [B] Examine the validity of the hypothesis and the conclusion of Lagrange's Mean Value theorem for the function $f(x) = 2x^2 7x + 10$ on [2, 5]
- **Q: 3** [A] If a refinement P^* of a partition P contains p points more than the points in P and $|f| \leq k, \forall x \in [a, b]$ then prove that $L(P, f) \leq L(P^*, f) \leq L(P, f) + 2pk\mu$

6



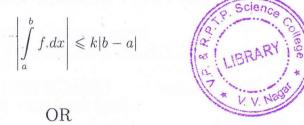
4

4

4

4

[B] If f is a bounded and integrable function on [a, b] and k is a number such that $|f| \leq k, \forall x \in [a, b]$ then prove that



4

4

4

8

Q: 3 [A] If f is bounded on [a, b] then for any two partitions P_1 and P_2 of [a, b], prove that $L(P_1, f) < U(P_2, f)$

[**B**] Show that a constant function k is integrable and evaluate $\int k dx$

Q: 4. Define Riemann Sum. Also show that a function f is integrable over [a, b] iff for $\epsilon > 0$, there exists $\delta > 0$ such that if P, P' are any two partitions of [a, b] with mesh less than δ then

$$|S(P,f) - S(P',f)| < \epsilon$$

OR

Q: 4 [A] Show that every continuous function is integrable 4

[B] Prove that a bounded function f having a finite number of points of discontinuty on [a, b] is integrable on [a, b].



2