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V.P. & R.P.T.P. Science College, V.V.Nagar

Internal Test: 2017-18

Subject : Mathematics US04CMTH02 Max. Marks : 25

Differential Equations

Date: 17/03/2018 Timing: 3.00 pm - 04.30 pm

Q: 1. Answer the following by choosing correct answers from given choices. **3**

[1] Integral curve of $2xdx = dy = 2zdz$ is given by

[A] $x^2 + y = c_1, y + z^2 = c_2$ [B] $x^2 + y = c_1, y - z^2 = c_2$
[C] $x^2 + y = c_1, y + z^2 = c_2$ [D] $x^2 - y = c_1, y - z^2 = c_2$

[2] A necessary and sufficient condition that there exists, between two functions $u(x, y)$ and $v(x, y)$ a relation $F(u, v) = 0$ not involving x and y explicitly is that

[A] $\frac{\partial(u,v)}{\partial(x,y)} = 0$ [B] $\frac{\partial(x,y)}{\partial(u,v)} = 0$ [C] $\frac{\partial(u,v)}{\partial(x,y)} \neq 0$ [D] $\frac{\partial(x,y)}{\partial(u,v)} \neq 0$

[3] Integral surface of the linear partial differential equation $p - qy = z^2$ can be obtained by solving the differential equation

[A] $dx = -\frac{dy}{y} = \frac{dz}{z^2}$ [B] $dx = \frac{dy}{y} = \frac{dz}{z^2}$
[C] $\frac{dx}{x} = -\frac{dy}{y} = -\frac{dz}{z^2}$ [D] $\frac{dx}{x} = \frac{dy}{y} = \frac{dz}{z^2}$

Q: 2. Answer any TWO of the following. **4**

[1] Find the integral curves of the equations $\frac{dx}{1+x} = \frac{dy}{1+y} = \frac{dz}{z}$

[2] Determine whether the equation $ydx + xdy = 5zdz$ is integrable or not.

[3] Obtain integral curve of the linear partial differential equation $px + qy^2 = z^3$

Q: 3 [A] Solve : $\frac{dx}{2xz} = \frac{dy}{2yz} = \frac{dz}{z^2 - x^2 - y^2}$ **3**

[B] Find the orthogonal trajectories on the conicoid $(x + y)z = 1$ of the conics in which it is cut by the system of planes $x - y + z = k$ **3**

OR

Q: 3 [A] Solve : $\frac{dx}{x} = \frac{dy}{y} = \frac{dz}{z - a\sqrt{x^2 + y^2 + z^2}}$ **3**

[B] Solve : $\frac{dx}{x^2} = \frac{dy}{y^2} = \frac{dz}{nxy}$ **3**

Q: 4 [A] Define Integrating Factor and prove that a Pfaffian differential equation in two variables always possesses an integrating factor **3**

[B] Solve : $x^2\frac{\partial z}{\partial x} + y^2\frac{\partial z}{\partial y} = (x+y)z$ **3**

OR

Q: 4 [A] If $f(u,v) = 0$ is a relation between u and v, where u and v are functions of x, y, z and z is a function of x and y then prove that partial differential equation of the relation is given by

$$\frac{\partial(u,v)}{\partial(y,z)}p + \frac{\partial(u,v)}{\partial(z,x)}q = \frac{\partial(u,v)}{\partial(x,y)}$$

3

[B] Determine whether the Pfaffian differential equation $yzdx+2xzdy-3xydz = 0$ is integrable or not. Find its solution if it is integrable **3**

Q: 5 [A] Show that $(x-a)^2+(y-b)^2+z^2 = 1$ is the complete integral of the non-linear partial differential equation $z^2(1+p^2+q^2) = 1$. Determine a general solution by finding the envelope of its particular solution. **3**

[B] Find the integral surface of the equation $x^2p+y^2q = -z^2$ which passes through the hyperbola $xy = x+y, \quad z = 1$ **3**

OR

Q: 5. Verify $z^2+\mu = 2(1+\lambda^{-1})(x+\lambda y)$ is a complete integral of partial differential equation $z = \frac{1}{p} + \frac{1}{q}$. Also show that the complete integral is the envelope of the one-parameter subsystem obtained by taking $b = -\frac{a}{\lambda} - \frac{\mu}{1+\lambda}$ in the solution $z = \sqrt{2x+a} + \sqrt{2y+b}$ of the differential equation. **6**

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