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V.P.& R.P.T.P.Science College , Vallabh Vidyanagar.
B.Sc. (Semester - III) Internal Test
US03CMTH01 (Advanced Calculus)

Date. 6/10/2017 ; Friday 3.00 p.m. to 4.30 p.m. Maximum Marks: 25

Que.1 Fill in the blanks. 3

(1) For $x + y = u$, $x - 2y = v$, jacobian $J =$ ....
(a) $\frac{1}{3}$ (b) $\frac{-1}{3}$ (c) $\frac{1}{2}$ (d) $\frac{-1}{2}$

(2) Area of plane region $r = a(1 + \cos\theta)$ is A = ...
(a) $\frac{3\pi a^2}{2}$ (b) $\frac{3\pi a}{2}$ (c) $\frac{\pi a^2}{2}$ (d) $\frac{3a^2}{2}$

(3) If $\bar{r} = u\,\bar{i} + v\,\bar{j} + uv\,\bar{k}$ then $EG - F^2 =$ ....
(a) $1 + v^2$ (b) uv (c) $1 + v^2 + u^2$ (d) $1 + u^2$

Que.2 Answer the following (Any Two) 4

(1) Evaluate $\int_0^1 \int_0^{\sqrt{1-x^2}} (x^2 + y^2)\, dydx$.

(2) Evaluate the line integral $\int_{(0,0,1)}^{(1,\pi/4,2)} [2xyz^2 dx + (x^2z^2 + z\cos yz)dy + (2x^2yz + y\cos yz)dz]$ on any path .

(3) Prove first fundamental form of a surface in cartesian form .

Que.3 (a) Transform $\iint_R (x-y)^2 \sin^2(x+y)\, dxdy$ in uv-plane by taking $x - y = u, x + y = v$.
Then evaluate it, where R : Parallelogram with vertices $(\pi, 0), (2\pi, \pi), (\pi, 2\pi), (0, \pi)$. 4

(b) Find area of the region bounded by $y = x^2$ *and* $y = 2x + 3$. 2

OR

Que.3 (a) Find the centroid of density 1 in the plane area bounded by $y = 2x - x^2$ and $y = 3x^2 - 6x$. 4

(b) Evaluate $\int_C 3(x^2 + y^2)ds$, where 2

C : Over the path y = -x from (-1,1) to (1,-1) (counterclockwise direction) .

Que.4 (a) State and prove Green's theorem for plane . 3

(b) Verify the result $\iint_R (\nabla \times \overline{V}) \cdot \bar{k}\ dxdy = \int_C \overline{V} \cdot \bar{u}\ ds$ for 3

$\overline{V} = y\bar{i} + 4x\bar{j}$ C : the boundary of triangle with vertices $(0,0), (2,0), (2,1)$

OR

Que.4 (a) Change the order of integration in $\int_0^{2a} \int_{\sqrt{2ax-x^2}}^{\sqrt{2ax}} f(x,y)\, dydx$. 4

(b) Prove that area of plane region in polar form are given by $A = \frac{1}{2}\int_C r^2 d\theta$. 2

Que.5 (a) State and prove divergence theorem of Gauss . 4

(b) By using divergence theorem , evaluate $\iint_S [x^3dydz + x^2ydzdx + x^2zdxdy]$,
where S: closed surface bounded by the plane z = 0 , z = b , $x^2 + y^2 = a^2$. 2

OR

Que.5 (a) Evaluate $\iint_S f(x,y,z)\, dA$, where $f(x,y,z) = xy$ and S : $z = xy$, $0 \le x$, $y \le 1$.

(b) In usual notation find $\sqrt{EG - F^2}$ for surface $\bar{r} = (a + b\cos v)(\cos u\bar{i} + \sin u\bar{j}) + b\sin v\bar{k}$. 2

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