V.P. & R.P.T.P. Science College, V.V.Nagar

Internal Test: 2016-17

Subject : Mathematics US04CMTH02 Max. Marks : 25

Differential Equations

Date: 11/03/2017 Timing: 3.00 pm - 04.30 pm

Q: 1. Answer the following by choosing correct answers from given choices. 3

[1] Integral curve of $\frac{dx}{x} = \frac{dy}{y} = \frac{dz}{z}$ is given by

[A] $x = c_1 y$; $y = c_2 z$ [B] $yz = c_1 x$; $xz = c_2 y$

[C] $yz + xz + xy = 0$ [D] $x + y + z = 0$

[2] A Paffian differential equation in two variables

[A] has no solution

[B] can have solution if certain conditions are satisfied

[C] always possesses a solution

[D] none of these

[3] A surface orthogonal to given system of surfaces cuts them at an angle measuring

[A] π [B] $\frac{\pi}{2}$ [C] $\frac{\pi}{3}$ [D] $\frac{\pi}{6}$

Q: 2. Answer any TWO of the following. 4

[1] Find the integral curves of the equations $\frac{dx}{1+x} = \frac{dy}{1+y} = \frac{dz}{z}$

[2] Determine whether the equation $ydx + xdy = 5zdz$ is integrable or not.

[3] Obtain a differential equation of the form $\frac{dx}{P} = \frac{dy}{Q} = \frac{dy}{R}$ whose integral curves generate surfaces orthogonal to the surfaces $x^2 - 2y^2 - 4z^2 = c$

Q: 3 [A] Solve : $\frac{dx}{x(y-z)} = \frac{dy}{y(z-x)} = \frac{dz}{z(x-y)}$ 3

[B] Find the integral curves of the equations

$$\frac{dx}{y(x+y)+az} = \frac{dy}{x(x+y)-az} = \frac{dz}{z(x+y)}$$

3

OR

Q: 3 [A] Solve : $\frac{dx}{z} = \frac{dy}{-z} = \frac{dz}{z^2+(x+y)^2}$ 3

[B] Find the orthogonal trajectories on the cone $x^2 + y^2 = z^2 \tan^2 \alpha$ of its intersection with the family of planes parallel to $z = 0$ 3

Q: 4 [A] If X is a vector such that $X.curlX = 0$ and μ is an arbitrary function of x, y and z then prove that $(\mu X).curl(\mu X) = 0$ 3

[B] Solve : $x^2 \dfrac{\partial z}{\partial x} + y^2 \dfrac{\partial z}{\partial y} = (x + y)z$ 3

OR

Q: 4 [A] Prove that the general solution of the linear differential equation $pP + qQ = R$ is $F(u, v) = 0$, where $F(u, v) = 0$ is an arbitrary function of $u(x, y, z) = c_1$ and $v(x, y, z) = c_2$ which form a solution of the equation $\dfrac{dx}{P} = \dfrac{dy}{Q} = \dfrac{dz}{R}$ 3

[B] Determine whether the Pfaffian differential equation

$$(y + z)dx + (z + x)dy + (x + y)dz = 0$$

is integrable or not. Find its solution if it is integrable 3

Q: 5. Find the integral surface of the linear partial differential equation $2y(z-3)p + (2x-z)q = y(2x-3)$ passing through the circle $z = 0, x^2 + y^2 = 2x$ 6

OR

Q: 5. Verify $z^2 + \mu = 2(1 + \lambda^{-1})(x + \lambda y)$ is a complete integral of partial differential equation $z = \dfrac{1}{p} + \dfrac{1}{q}$. Also show that the complete integral is the envelope of the one-parameter subsystem obtained by taking $b = -\dfrac{a}{\lambda} - \dfrac{\mu}{1 + \lambda}$ in the solution $z = \sqrt{2x + a} + \sqrt{2y + b}$ of the differential equation. 6