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V.P.& R.P.T.P.Science College , Vallabh Vidyanagar.
B.Sc. (Semester - III) Internal Test
US03CMTH01 (Advanced Calculus)

Date. 3/10/2016 ; Monday 3.00 p.m. to 4.30 p.m. Maximum Marks: 25

Que.1 Fill in the blanks. 3

(1) $\int_0^2 \int_0^x dydx =$ ....
(a) 1 (b) 1/2 (c) x (d) 2

(2) In double integral , Total mass M of density 1 over region $1 \le x, y \le 2$ is ....
(a) 1 (b) 2 (c) 0 (d) 4

(3) Area of plane region in Polar form is given by A = ...
(a) $\frac{1}{2}\int_C r^2 d\theta$ (b) $\int_C r^2 d\theta$ (c) $\frac{1}{2}\int_C r\, d\theta$ (d) $\frac{1}{2}\int_C [x\, dx - y\, dy]$

Que.2 Answer the following (Any Two) 4

(1) Evaluate $\int_C [y^2 dx - x^2 dy]$, where C : along the circle $x^2 + y^2 = 1$ from (0,1) to (1,0) (counterclockwise direction).

(2) Find area of region in the first quadrant bounded by $y = x$, $y = x^3$.

(3) In usual notation find $\sqrt{EG - F^2}$, for surface $\overline{r} = (a + b\cos v)(\cos u\overline{i} + \sin u\overline{j}) + b\sin v\overline{k}$.

Que.3 (a) Transform $\iint_R (x+y)^3\, dxdy$ in uv-plane by taking $x + y = u, x - 2y = v$.
Then evaluate it, where R : Parallelogram with vertices $(1,0), (0,1), (3,1), (2,2)$. 4

(b) Evaluate $\iint_R e^{-x^2-y^2}\, dxdy$ where $R : x^2 + y^2 \le 1$. 2

OR

Que.3 (a) Find volume of the the tetrahedron cut from the first octant by the plane $3x + 4y + 2z = 12$. 3

(b) Find the co-ordinate $\bar{x}$ of centroid of density 1 in the plane area bounded by $y = 6x - x^2$ and $y = x$. 3

Que.4 (a) State and prove Green's theorem for plane . 4

(b) Evaluate $\int_C [y^3\, dx + (x^3 + 3y^2 x)\, dy]$ by using Green's theorem , where
C : the boundary of region bounded by $y = x^2$ and y = x (counterclockwise direction). 2

OR

Que.4 (a) Change the order of integration in $\int_0^{a/2} \int_{x^2/a}^{x - x^2/a} f(x,y)\, dydx$. 3

(b) Verify the result $\iint_R (\nabla \times \overline{V}) \cdot \overline{k}\ dxdy = \int_C \overline{V} \cdot \overline{u}\ ds$.
for $\overline{V} = y\overline{i} + 4x\overline{j}$; $C :$ the boundary of triangle with vertices $(0,0), (2,0), (2,1)$. 3

Que.5 (a) State and prove divergence theorem of Gauss . 4

(b) By using divergence theorem , evaluate $\iint_S [2z(xy - x - y)dxdy + x^2 dydz + y^2 dzdx]$,
where S : The surface of cube $0 \le x\ , \ y\ , \ z \le 1$. 2

OR

Que.5 (a) Find area of the surface $z = x^2 + y^2$, where $0 \le z \le b$. 3

(b) Evaluate $\iint_S f(x,y,z)\, dA$, where $f(x,y,z) = (x^2 + y^2)^2$.
$S : z = (x^2 + y^2)^2, x^2 + y^2 \le 1$ 3

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