# V.P. \& R.P.T.P.Science College, V.V.Nagar Internal Test <br> B.Sc.SEMISTER -IV <br> Subject: Mathematics (US04EMTH05) 

Date: 15/03/2014
Day : Saturday

Maximum Marks:30
Time : 1.00 pm to 2.00 pm

Que. 1 Attempt the following.(Any Three)

1. Find normal vector of the function $y^{2}=2 x^{3}$ at point $(2,4)$.
2. Prove that $\bar{\nabla}\left(\frac{f}{g}\right)=\frac{g \bar{\nabla} f-f \bar{\nabla} g}{g^{2}}$
3. Prove that $\bar{\nabla} \cdot(f \bar{\nabla} g)=f \nabla^{2} g+\bar{\nabla} f \cdot \bar{\nabla} g$
4. Define Curl of vector field.

5. Prove that $\forall a \in \mathbb{B}$, (1) $a+a=a$ (2) $a \cdot a=a$
6. If $a+x=b+x$ and $a+x^{\prime}=b+x^{\prime}$ then prove that $a=b$

Que. 2 [A] Prove that $f(x, y)=\tan ^{-1}\left(\frac{y}{x}\right)$ is harmonic function
[B] Find Gradient of $f(x, y)=\log r$, where $\bar{r}=x \bar{i}+y \bar{j}$

OR
Que. 2 [A] Find directional derivative of $f(x, y, z)=\left(2 x^{2}+3 y^{2}+z^{2}\right)$ at point $(2,1,3)$ in direction of $\bar{a}=\bar{i}-2 \bar{k}$
[B] Find Gradient of $f(x, y, z)=\left(x^{2}+y^{2}+z^{2}\right)^{2}$ at point $(1,2,3)$

Que. 3 [C] Verify $\bar{\nabla} \cdot(f \bar{v})=f(\bar{\nabla} \cdot \bar{v})+\bar{v} \cdot(\bar{\nabla} f)$ for $f=e^{x y z}$ and $\bar{v}=a x \bar{i}+b y \bar{j}+c z \bar{k}$
[D] Prove that $\bar{\nabla} \cdot(\bar{\nabla} \times \bar{v})=0$

## OR

Que. 3 [C] If $f(x, y)=\log \left(x^{2}+y^{2}\right)$ then prove that $\bar{\nabla} \cdot(\bar{\nabla} f)=\nabla^{2} f=0$
[D] Prove that $\bar{\nabla} \cdot\left(r^{n} \bar{r}\right)=(n+3) r^{n}$ where $\bar{r}=x \bar{i}+y \bar{j}+z \bar{k} ; r=|\bar{r}|$
Que. 4 [E] State and prove De-Morgan's law for Boolean algebra.
[F] If $a$ and $b$ are elements of a Boolean algebra $\mathbb{B}$ satisfying the relation $a \leq b$ then prove that $a+b c=b(a+c), \forall c \in \mathbb{B}$

Que. 4 [E] Find Boolean function of given switching circuit , then simplified it. Also draw simplified circuit.

$[F] \forall a \in \mathbb{B} ;$ prove that inverse of $a$ is unique.


