V.P. & R.P.T.P. Science College, V.V.Nagar

Internal Test: 2017-18

Subject : Mathematics US02CMTH02 Max. Marks : 25

Matrix Algebra and Differential Equations

Date: 20/03/2018 Timing: 01:30 pm - 02:30 pm

Q: 1. Answer the following by choosing correct answers from given choices. **3**

[1] Matrix $A = \begin{bmatrix} 3 & -2 & 4 \\ -2 & 6 & 0 \\ 4 & 0 & 1 \end{bmatrix}$ is a

[A] scalar matrix [B] diagonal matrix
[C] symmetric matrix [D] skew-symmetric matrix

[2] Matrix $P = \begin{bmatrix} 6 & -7 \\ 12 & -14 \end{bmatrix}$ is

[A] orthogonal [B] singular [C] non-singular [D] none

[3] The Complementary Function of $(D^2 + 4)y = X$ is

[A] $c_1 \cos \sqrt{2}x + c_2 \sin \sqrt{2}x$ [B] $c_1 \cos 2x + c_2 \sin 2x$
[C] $c_1 e^{2x} + c_2 e^{-2x}$ [D] $c_1 e^{4x} + c_2 e^{-4x}$

Q: 2. Answer any TWO of the following. **4**

[1] Define : (i) Transpose of a matrix (ii) Identity Matrix

[2] Find the characteristic equation of $\begin{bmatrix} 2 & 4 \\ 1 & -5 \end{bmatrix}$.

[3] Find $\frac{1}{(D+2)^3} e^{-2x}$

Q: 3 [A] Prove that every Hermitian matrix over $\mathbb{C}$ can be uniquely expressed as $P+iQ$, where P and Q are real symmetric and skew-symmetric matrices respectively **3**

[B] If $A = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$ then show that $A^k = \begin{bmatrix} 1+2k & -4k \\ k & 1-2k \end{bmatrix}$ where k is any positive integer **3**

OR

Q: 3 [A] State and prove the *reversal law* for the transpose of product of matrices and deduce the reversal law for conjugate transpose of product of matrices. **3**

[B] State and prove associative law for product of matrices **3**

Q: 4 [A] If S is a real skew-symmetric matrix then prove that $I - S$ is non-singular and the matrix $A = (I+S)(I-S)^{-1}$ is orthogonal **3**

[B] Find the characteristic equation of the matrix $A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$ and verify that it is satisfied by A and hence obtain A^{-1} **3**

OR

Q: 4 [A] State and prove *Cayley-Hamilton theorem* **3**

[B] Find the characteristic roots and any one of the characteristic vectors of : $\begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$ **3**

Q: 5 [A] Obtain the rule for finding the particular integral of $f(D)y = e^{mx}$ where m is a constant **3**

[B] Solve : $(D^3 - 5D^2 + 7D - 3)y = \cosh x$ **3**

OR

Q: 5 [A] Solve : $(D^2 + a^2)y = cosecax$ **3**

[B] Solve $(D^2 - 5D + 6)y = 4e^x$ subject to the conditions that $y(0) = y'(0) = 1$. Hence find y(16) **3**