

V.P. & R.P.T.P. Science College, V.V.Nagar

Internal Test: 2017-18

Subject : Mathematics US02CMTH02 Max. Marks : 25

Matrix Algebra and Differential Equations

Date: 20/03/2018

Timing: 01:30 pm - 02:30 pm

Q: 1. Answer the following by choosing correct answers from given choices.

3

[1] Matrix $A = \begin{bmatrix} 3 & -2 & 4 \\ -2 & 6 & 0 \\ 4 & 0 & 1 \end{bmatrix}$ is a

[A] scalar matrix
[C] symmetric matrix

[B] diagonal matrix
[D] skew-symmetric matrix

[2] Matrix $P = \begin{bmatrix} 6 & -7 \\ 12 & -14 \end{bmatrix}$ is

[A] orthogonal [B] singular

[C] non-singular [D] none

[3] The Complementary Function of $(D^2 + 4)y = X$ is

[A] $c_1 \cos \sqrt{2}x + c_2 \sin \sqrt{2}x$

[B] $c_1 \cos 2x + c_2 \sin 2x$

[C] $c_1 e^{2x} + c_2 e^{-2x}$

[D] $c_1 e^{4x} + c_2 e^{-4x}$

Q: 2. Answer any TWO of the following.

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[1] Define : (i) Transpose of a matrix (ii) Identity Matrix

[2] Find the characteristic equation of $\begin{bmatrix} 2 & 4 \\ 1 & -5 \end{bmatrix}$.

[3] Find $\frac{1}{(D+2)^3} e^{-2x}$

Q: 3 [A] Prove that every Hermitian matrix over \mathbb{C} can be uniquely expressed as $P+iQ$, where P and Q are real symmetric and skew-symmetric matrices respectively

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[B] If $A = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$ then show that $A^k = \begin{bmatrix} 1+2k & -4k \\ k & 1-2k \end{bmatrix}$ where k is any positive integer

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OR

Q: 3 [A] State and prove the reversal law for the transpose of product of matrices and deduce the reversal law for conjugate transpose of product of matrices.

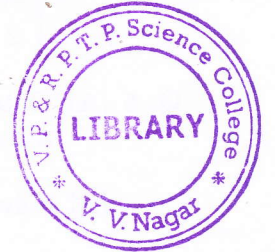
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[B] State and prove associative law for product of matrices

3

Q: 4 [A] If S is a real skew-symmetric matrix then prove that $I - S$ is non-singular and the matrix $A = (I + S)(I - S)^{-1}$ is orthogonal

3



[B] Find the characteristic equation of the matrix $A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$ and verify that it is satisfied by A and hence obtain A^{-1} 3

OR

Q: 4 [A] State and prove *Cayley-Hamilton theorem* 3

[B] Find the characteristic roots and any one of the characteristic vectors of : 3

$$\begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$$

Q: 5 [A] Obtain the rule for finding the particular integral of $f(D)y = e^{mx}$ where m is a constant 3

[B] Solve : $(D^3 - 5D^2 + 7D - 3)y = \cosh x$ 3

OR

Q: 5 [A] Solve : $(D^2 + a^2)y = \operatorname{cosec} ax$ 3

[B] Solve $(D^2 - 5D + 6)y = 4e^x$ subject to the conditions that $y(0) = y'(0) = 1$. Hence find $y(16)$ 3

