V.P. & R.P.T.P. Science College, V.V.Nagar

Internal Test: 2017-18

Subject : Mathematics US01CMTH02 Max. Marks : 25

Calculus and Differential Equation

Date: 11/10/2017 Timing: 01:30 pm - 02:30 pm

Q: 1. Answer the following by choosing correct answers from given choices. **3**

[1] If $y = e^{3x}\cos 2x$ then $y_n =$

[A] $13^{\frac{n}{2}}e^{3x}\cos(2x + n\tan^{-1}\frac{1}{3})$ [B] $13^{\frac{n}{2}}\, e^{3x}\cos(2x + n\tan^{-1}\frac{3}{2})$

[C] $13^{\frac{n}{2}}e^{3x}\cos(2x + n\tan^{-1}3)$ [D] $13^{\frac{n}{2}}e^{3x}\cos(2x + n\tan^{-1}\frac{2}{3})$

[2] Intrinsic equation of a curve involves

[A] cartesian coordinates only [B] polar coordinates only

[C] parameteric cordinates only [D] none of these

[3] If a function y of x be implicitly described by $f(x, y) = c$, where c is a constant then

[A] $\frac{dy}{dx} = -\frac{f_y}{f_x}$ [B] $\frac{dy}{dx} = \frac{f_y}{f_x}$ [C] $\frac{dy}{dx} = \frac{f_x}{f_y}$ [D] $\frac{dy}{dx} = -\frac{f_x}{f_y}$

Q: 2. Answer ANY TWO of the following. **4**

[1] If $y = \log(2x - 1)$ then find y_4

[2] Define : (i) Intrinsic Equation (ii) Rectification

[3] Verify Euler's theorem for the function $z = \sin^{-1}\frac{x^2}{y^2}$

Q: 3 [A] State and prove Leibniz's theorem **3**

[B] If $y = e^{ax}\cos(bx + c)$, then prove that $y_n = r^n e^{ax}\cos(bx + c + n\varphi)$, where $r = \sqrt{a^2 + b^2}$, $\varphi = \tan^{-1}\left(\frac{b}{a}\right)$ **3**

OR

Q: 3 [A] Let $y = (x^2 - 2)^m$. Find the value of m such that $(x^2 - 2)y_{n+2} + 2xy_{n+1} - n(n + 1)y_n = 0$. **3**

[B] Find the angle between radius vector and tangent at a point on the curve $r^m = a^m(\cos m\theta + \sin m\theta)$ **3**

Q: 4 [A] Prove that if ρ is the radius of curvature at any point P of the parabola $y^2 = 4ax$ and S is its focus then prove that $\rho^2 \propto SP^3$ **3**

[B] For the curve $r = a(1 - \cos\theta)$, prove that $\rho^2 \propto r$. Also prove that if ρ_1 and ρ_2 are radii of the curvature at the ends of a chord through the pole, $\rho_1^2 + \rho_2^2 = \frac{16a^2}{9}$ **3**

OR

Q: 4. Define Radius of curvature and let $r = f(\theta)$ be a polar form of a curve with a point P on it. Then prove that the radius of curvature at P is given by

$$\rho = \frac{(r^2 + r_1^2)^{3/2}}{r^2 + 2r_1^2 - rr_2},$$

where $r_1 = f'(\theta)$ and $r_2 = f''(\theta)$ **6**

Q: 5 [A] Let $z = f(x, y)$ be a real valued function defined on $E \subset R^2$. Suppose that f is a homogeneous function of degree n and that all the second order partial derivatives of f exist and are continuous. Then prove that

$$x^2\frac{\partial^2 z}{\partial x^2} + 2xy\frac{\partial^2 z}{\partial x \partial y} + y^2\frac{\partial^2 z}{\partial y^2} = n(n-1)z.$$

3

[B] If $H = f(2x - 3y, 3y - 4z, 4z - 2x)$, then prove that

$$\frac{1}{2}\frac{\partial H}{\partial x} + \frac{1}{3}\frac{\partial H}{\partial y} + \frac{1}{4}\frac{\partial H}{\partial z} = 0.$$

3

OR

Q: 5 [A] Let a function y of x be implicitly described by $f(x, y) = c$. Then prove that

(1) $\dfrac{dy}{dx} = -\dfrac{f_x}{f_y}$

(2) $\dfrac{d^2y}{dx^2} = -\dfrac{f_{xx}(f_y)^2 - 2f_{xy}f_xf_y + f_{yy}(f_x)^2}{(f_y)^3}$ **3**

[B] If A, B and C are angles of a ΔABC such that $\sin^2 A + \sin^2 B + \sin^2 C = K$, a constant, then prove that

$$\frac{dB}{dC} = \frac{\tan C - \tan A}{\tan A - \tan B}$$

3