V.P. & R.P.T.P. Science College,V.V.Nagar

Internal Test: 2016-17

Subject : Mathematics US02CMTH02 Max. Marks : 25

Matrix Algebra and Differential Equations

Date: 15/03/2017 Timing: 01:30 pm - 02:30 pm

Q: 1. Answer the following by choosing correct answers from given choices. **3**

[1] Two matrices can be added only if they have same

[A] orders [B] number of columns [C] number of rows [D] elements

[2] If $|A + 4I| = 0$ then one of the characteristic roots of A is

[A] 0 [B] -4 [C] 4 [D] 1

[3] $\frac{1}{D^2}e^x =$

[A] $\frac{1}{2!}e^x$ [B] e^x [C] $2e^x$ [D] $\frac{1}{e^{2x}}$

Q: 2. Answer any TWO of the following. **4**

[1] Define : (i) Conjugate Transpose (ii) Skew-symmetric Matrix

[2] Verify $A^2 - 8A - I = O$, where $A = \begin{bmatrix} 7 & 2 \\ 4 & 1 \end{bmatrix}$.

[3] Find $\frac{1}{(D^6 + D^2 + 1)} \sin 2x$

Q: 3 [A] Prove that every square matrix can be expressed in one and only one way as a sum of a symmetric and a skew-symmetric matrix. **3**

[B] For $A = \begin{bmatrix} 0 & 2m & n \\ l & m & -n \\ l & -m & n \end{bmatrix}$, where $l = \frac{1}{\sqrt{2}}$, $m = \frac{1}{\sqrt{6}}$ and $n = \frac{1}{\sqrt{3}}$ show that $AA' = I$ **3**

OR

Q: 3 [A] State and prove the *reversal law* for the transpose of product of matrices and deduce the reversal law for conjugate transpose of product of matrices. **3**

[B] If $A = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$ then show that $A^k = \begin{bmatrix} 1 + 2k & -4k \\ k & 1 - 2k \end{bmatrix}$ where k is any positive integer **3**

Q: 4 [A] State and prove *Cayley-Hamilton theorem* **3**

[B] Find eigen values and any one of the eigen vectors of $\begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$ **3**

OR

Q: 4 [A] Prove that every orthogonal matrix A can be expressed as $A = (I+S)(I-S)^{-1}$ by a suitable choice of real skew-symmetric matrix S provided that -1 is not a characteristic root of A **3**

[B] Find the characteristic equation of the matrix $A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$ and verify that it is satisfied by A and hence obtain A^{-1} **3**

Q: 5 [A] Obtain the rule for finding the particular integral of $f(D)y = e^{mx}$ where m is a constant **3**

[B] Solve : $(D^2 + 4)y = \sec 2x$ **3**

OR

Q: 5 [A] In usual notations prove that $\frac{1}{D-\alpha}X = e^{ax}\int Xe^{-ax}.dx$ **3**

[B] Solve : $\frac{d^4y}{dx^4} - 2\frac{d^3y}{dx^3} + 5\frac{d^2y}{dx^2} - 8\frac{dy}{dx} + 4y = 0$ **3**