# V.P. & R.P.T.P. Science College, V.V.Nagar

Internal Test: 2016-17

Subject : Mathematics US01CMTH02 Max. Marks : 25

Calculus and Differential Equations

Date: 07/10/2016 Timing: 01:30 pm - 02:30 pm

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**Q: 1.** Answer the following by choosing correct answers from given choices. 3

[ 1] If $y = e^{3x}\cos 2x$ then $y_n =$

[A] $13^{\frac{n}{2}} e^{3x}\cos(2x + n\tan^{-1}\frac{1}{3})$ [B] $13^{\frac{n}{2}} e^{3x}\cos(2x + n\tan^{-1}\frac{3}{2})$

[C] $13^{\frac{n}{2}} e^{3x}\cos(2x + n\tan^{-1} 3)$ [D] $13^{\frac{n}{2}} e^{3x}\cos(2x + n\tan^{-1}\frac{2}{3})$

[ 2] For a polar curve $r = f(\theta)$ the radius of curvature at a point $(r, \theta)$ is given by

[A] $\sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}$ [B] $\frac{(r^2 + {r_1}^2)^{\frac{3}{2}}}{r^2 + 2{r_1}^2 - rr_2}$ [C] $\sqrt{1 + \left(\frac{dr}{d\theta}\right)^2}$ [D] $\frac{({r_1}^2 + {r_2}^2)^{\frac{3}{2}}}{r^2 + 2{r_1}^2 - rr_2}$

[ 3] Degree of a homogeneous function defined by $f(x, y) = \dfrac{\sqrt[3]{x} - \sqrt[3]{y}}{x + y}$ is

[A] $-\frac{3}{2}$ [B] $\frac{3}{2}$ [C] $\frac{2}{3}$ [D] $-\frac{2}{3}$

**Q: 2.** Answer any TWO of the following. 4

[ 1] If $y = e^{2x}\sin 5x$ then find $y_4$

[ 2] Define : (i) Radius of Curvature (ii) Rectification

[ 3] Verify Euler's theorem for the function $z = x^2y - xy^2$

**Q: 3 [A]** State and prove Leibniz's theorem 3

[ B] If $y = \sin(ax + b)$, then prove that $y_n = a^n \sin\left(ax + b + \frac{n\pi}{2}\right)$ 3

OR

**Q: 3 [A]** Find the angle between radius vector and tangent at a point on the curve $r^m = a^m(\cos m\theta + \sin m\theta)$ 3

[ B] If $y = e^{ax}\sin(bx + c)$, then prove that $y_n = r^n e^{ax}\sin(bx + c + n\varphi)$, where $r = \sqrt{a^2 + b^2}$ and $\varphi = \tan^{-1}\left(\frac{b}{a}\right)$ 3

**Q: 4.** Define radius of curvature. Let $r = f(\theta)$ be a polar form of a curve with a point $P$ on it. Then prove that the radius of curvature at $P$ is given by

$$\rho = \frac{(r^2 + r_1^2)^{3/2}}{r^2 + 2r_1^2 - rr_2},$$

where $r_1 = f'(\theta)$ and $r_2 = f''(\theta)$ 6

OR

Q: 4 [A] In usual notations prove that $\frac{ds}{dx} = \sqrt{1 + \left(\frac{dy}{dx}\right)^2}$ 3

[ B] Find the entire length of the astroid $x^{2/3} + y^{2/3} = a^{2/3}$ 3

Q: 5 [A] State and prove the Euler's theorem for functions of two variables. 3

[ B] If $H = f(2x - 3y, 3y - 4z, 4z - 2x)$, then prove that

$$\frac{1}{2}\frac{\partial H}{\partial x} + \frac{1}{3}\frac{\partial H}{\partial y} + \frac{1}{4}\frac{\partial H}{\partial z} = 0.$$

3



OR

Q: 5 [A] If $u = \sin^{-1}\left(\frac{x^2y^2}{x+y}\right)$, then prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 3\tan u$

3

[ B] If $z = f(x, y)$, $x = r\cos\theta$, $y = r\sin\theta$, then prove that

$$\left[\frac{\partial z}{\partial x}\right]^2 + \left[\frac{\partial z}{\partial y}\right]^2 = \left[\frac{\partial z}{\partial r}\right]^2 + \frac{1}{r^2}\left[\frac{\partial z}{\partial \theta}\right]^2.$$

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