

V.P.&amp; R.P.T.P.Science College,Vallabh Vidyanagar.

Internal Test

B.Sc. Semester II

US02CMTH01 ( Analytic Solid Geometry )

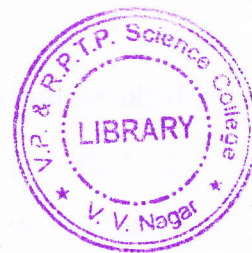
Saturday , 15<sup>th</sup> March 2014

11.00 a.m. to 12.00 p.m.

Maximum Marks: 30

Que.1 Answer the following ( Any three )

- (1) Prove that a sphere with centre  $(\alpha, \beta, \gamma)$  and radius  $a$  is given by  $(x - \alpha)^2 + (y - \beta)^2 + (z - \gamma)^2 = a^2$ .
- (2) Find the equations of the tangent plane to the sphere  $x^2 + y^2 + z^2 + 2x + 4y + 6z - 24 = 0$  at  $(1, 1, 2)$ .
- (3) Show that  $Ax^2 + By^2 + Cz^2 = D$  represents an elliptic hyperboloid of one sheet if one coefficient is negative and  $D > 0$ .
- (4) Plot the spherical point  $(2, 7\pi/4, \pi/6)$ .
- (5) Find the points of intersection of the line  $\frac{x - \alpha}{l} = \frac{y - \beta}{m} = \frac{z - \gamma}{n}$  and the cone  $f(x, y, z) \equiv ax^2 + by^2 + cz^2 + 2fyz + 2gzx + 2hxy + 2ux + 2vy + 2wz = 0$ .
- (6) Find the equation of cone with vertex at the origin and which passes through the curve  $ax^2 + by^2 = 2z$  ;  $lx + my + nz = p$ .



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- Que.2 (a) Find the equation of the spheres which pass through the given circle  $x^2 + y^2 + z^2 - 4x - y + 3z + 12 = 0$  ;  $2x + 3y - 7z = 10$  and touch the plane  $x - 2y + 2z = 1$  . 6
- (b) Find equation of sphere with centre at  $(2, -1, 0)$  and passing through  $(1, -1, 2)$ . 2

OR

- Que.2 (a) Let two spheres be given by 5
- $$S_1 \equiv x^2 + y^2 + z^2 + 2u_1x + 2v_1y + 2w_1z + d_1 = 0 ;$$
- $$S_2 \equiv x^2 + y^2 + z^2 + 2u_2x + 2v_2y + 2w_2z + d_2 = 0.$$

Then prove that  $S_1 + \lambda S_2 = 0$  , where  $\lambda \in \mathbb{R}$  ,  $\lambda \neq -1$ , represents a family of spheres passing through the intersection of the spheres  $S_1 = 0$  and  $S_2 = 0$  .

- (b) Show that the following pair of spheres touch each other 3
- $$x^2 + y^2 + z^2 = 64 ; x^2 + y^2 + z^2 - 12x + 4y - 6z + 48 = 0 .$$

- Que.3 (a) Identify , describe and sketch the surface  $\frac{x^2}{a^2} + \frac{y^2}{b^2} = cz$  ;  $(c > 0)$ . 5
- (b) Find Jacobian of Cartesian co-ordinates with respect to Cylindrical co-ordinates. 3

OR

Que.3 (a) By a proper choice of axes , prove that the Cartesian coordinates (  $x$  ,  $y$  ,  $z$  ) of a point can be expressed in terms of spherical polar coordinates  $(\rho, \theta, \phi)$  as  
 $x = \rho \sin \phi \cos \theta$  ,  $y = \rho \sin \phi \sin \theta$  ,  $z = \rho \cos \phi$  .

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(b) Identify and describe the surface  $\frac{x^2}{9} - \frac{y^2}{16} - \frac{z^2}{9} = 1$  .

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Que.4 (a) Prove that the cone  $ax^2 + by^2 + cz^2 + 2fyz + 2gzx + 2hxy = 0$  admits of sets of three mutually perpendicular generators iff  $a + b + c = 0$  .

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(b) Find the equation of cone whose vertex is  $(\alpha, \beta, \gamma)$  and base is  $ax^2 + by^2 = 1$  ;  $z = 0$  .

3

OR

Que.4 (a) If the section of a cone whose vertex is P and guiding curve is the ellipse

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$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$  ;  $z = 0$  by the plane  $x=0$  is a rectangular hyperbola .Show that the

locus of P is  $\frac{x^2}{a^2} + \frac{y^2 + z^2}{b^2} = 1$  .

(b) If  $F(x, y, z) \equiv ax^2 + by^2 + cz^2 + 2fyz + 2gzx + 2hxy + 2ux + 2vy + 2wz + d = 0$  represents a cone , then the co-ordinates of its vertex satisfy the equation  
 $F_x = F_y = F_z = F_t = 0$  where t is used to make  $F(x,y,z)$  homogeneous and put t equal to unity after differentiation.

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