TYBSc [Semester-6] Physics

US06CPHY23 Nuclear Physics

UNIT- 4 Part 1 Lecture 3

Detectors and Accelerators

Ch 6 Accelerators: Topics

6.1 Introduction
6.2 Cockcroft and Walton Generator
6.3 Van de Graff Accelerator
6.4 Tandem accelerator
6.5 Linear Accelerator or Drift Tube accelerator,
6.7 Magnetic resonance accelerators or cyclotron
6.8 Betatron
6.9 Synchrocyclotron or frequency modulated cyclotrons

Ch 6 Accelerators: Topics

Recommended Books:

Nuclear and Particle Physics (2nd edition)
V K Mittal, R C Verma and S C Gupta
PHI Learning Pvt. Ltd.

6.5 LINEAR ACCELERATOR (LINAC)

6.5.4 Advantages

1. Requirement of generating very high-voltages-million volts range is avoided in these accelerators.

2. They are **economical** for obtaining very high-energy particle beams.

3. They provide **well-collimated beam** of accelerated ions.

6.5 LINEAR ACCELERATOR (LINAC)

6.5.5 Limitations

1. They are inconveniently **long in size**.

2. They require **extremely high frequency** and **high-voltage oscillator**.

6.7 CYCLOTRONS

Advantages

1. Cyclotron is much **smaller in size** compared to linear accelerators.

2. No high voltages (like in Van de Graaff accelerator) are required. Only low-voltage ac oscillator (10-50 kV peak value) is required.

3. Cyclotron can deliver tens of microamperes of current at the target.

6.7 CYCLOTRONS

6.7.6 Limitations (1) Cost

- It has been estimated that the cost of building larger cyclotrons scales roughly as the cube of the energy.

- For example, the cost of **500 MeV** cyclotron is about **USS 108**.
- To build a cyclotron of **5 GeV** is beyond the means of most of the countries.

6.7 CYCLOTRONS

6.7.6 Limitations (2)

- As the energy of ions increases, **relativistic effects** come into picture.

$$t = \frac{\pi m}{B q} \qquad (6.5)$$

$$f = \frac{1}{T} = \frac{B q}{2 \pi m} \qquad (6.6)$$

6.7 CYCLOTRONS

6.7.6 Limitations (2)

- Two methods for this compensation are possible

6.7 CYCLOTRONS

- Method-I
- Change frequency keeping the magnetic field constant.

$$fm = \frac{B\,q}{2\,\pi}$$

- Cyclotrons based on this principle are known as Synchrocyclotrons or Frequency Modulated Cyclotron,

6.7 CYCLOTRONS

Method II

- Increase the magnetic field B in proportion to the mass, so that B/m remains constant.

$$t = \frac{\pi m}{B q}$$

- Due to this, the time period / in Eg. (6.5) remains unaffected, by the increase of mass.
- Accelerators based on this principle are known as Sector Focusing or ***Azimuthally Varying Field* (AVF) Cyclotrons.**

6.8 BETATRON

6.8 BETATRON

- It is an accelerator for accelerating **electrons** or **beta particles**.

- There are definite problems associated with accelerating electrons by Van de Graaff and Cyclotron accelerators.

- The former machine could accelerate electrons up to a few MeV while in case of latter machine relativistic effect becomes prominent.

6.8 BETATRON

- **D.W. Kerst** in 1940 built a new accelerator called betatron, which could accelerate electrons up to **250 MeV.**

6.8 BETATRON

6.8.1 Principle

- This machine uses the concept of electromagnetic induction as the accelerating force.

- It employs time varying magnetic field, which give rise to induced electric field on the electrons moving in a fixed circular orbit and these electrons are accelerated.

6.8 BETATRON

6.8.1 Principle

- The rate of increase of magnetic flux (ϕ) is very slow compared to the frequency with which electrons are orbiting.
- In each orbit electrons are accelerated.

6.8 BETATRON

6.8.2 Construction

- It consists of a doughnut-shaped chamber, which is placed between the pole pieces of an electromagnet.
- The chamber is highly evacuated and the electrons with certain kinetic energy are injected into a circular orbit by an electron gun.

6.8.2 Construction

- The electromagnet is powered by an alternating current.
- The inner layer of the chamber is coated with a thin layer of silver to avoid surface charge accumulation.

6.8 BETATRON

6.8.3 Working

- The electrons are injected in the doughnut of the betatron during the first quarter of a cycle in which magnetic field linking electron orbit is rising.

6.8 BETATRON

6.8.3 Working

- Now to have electrons in a fixed orbit of radius r_0, a relation between the magnetic field at the orbit B_0 and the total magnetic flux $\emptyset$ has to be derived

6.8 BETATRON

6.8.3 Working

- An ac current having a frequency in the range 60-100 Hz powers the electromagnets.
- This generates a slow-varying field at the electron orbit of fixed radius.

6.8 BETATRON

6.8.3 Working

- The momentum of the electron

$$p = r_0\, e\, B_0$$

- The force acting on the electron

$$\frac{d}{dt}(p) = \frac{d}{dt}(r_o\, e\, B_0) = r_o\, e\, \frac{d\, B_0}{dt} \qquad (6.10)$$

6.8 BETATRON

6.8.3 Working

- Induced e.m.f. (ε) in the electron orbit is equal to work done the unit charge in going around the orbit of radius r_0 i.e.

$$\varepsilon = \oint \boldsymbol{E}.\boldsymbol{dl}$$

- where E is the electric field which accelerates the electrons.

6.8 BETATRON

$$\varepsilon = \oint \boldsymbol{E.dl}$$

6.8.3 Working

- From Faraday's laws is given by rate of change of magnetic flux

$$\varepsilon = 2\,\pi\;r_o\,E\;=\frac{d\,\emptyset}{dt} \qquad (6.11)$$

- The force on the electron is

$$e\,E = \;\frac{d\,p}{dt} \qquad (6.12)$$

6.8 BETATRON

6.8.3 Working

$$\frac{d}{dt}(p) = r_o\, e\, \frac{d\,B_0}{dt} \qquad (6.10)$$

$$\varepsilon = 2\,\pi\; r_o\, E \;=\; \frac{d\,\emptyset}{dt} \qquad (6.11)$$

$$e\,E = \frac{d\,p}{dt} \qquad (6.12)$$

- Combining Eqs. (6.10), (6.11) and (6.12)

$$\frac{d\,\emptyset}{dt} = 2\,\pi\; r_o{}^{2}\; \frac{d\,B_0}{dt} \qquad (6.13)$$

6.8 BETATRON

6.8.3 Working

$$\frac{d}{dt}(p) = r_o\, e\, \frac{d\,B_0}{dt} \qquad (6.10)$$

$$\varepsilon = 2\pi\ r_o\, E\ = \frac{d\,\emptyset}{dt} \qquad (6.11)$$

$$e\,E = \frac{d\,p}{dt} \qquad (6.12)$$

- $\frac{d\,\emptyset}{dt} = 2\pi\ r_o\, E\ = 2\pi\ r_o\left(\frac{1}{e}\,\frac{d\,p}{dt}\right) = 2\pi\ r_o\left(\frac{1}{e}\right)\left\{r_o\, e\, \frac{d\,B_0}{dt}\right\}$

$$\frac{d\,\emptyset}{dt} = 2\pi\ r_o{}^2\ \frac{d\,B_0}{dt} \qquad (6.13)$$

6.8 BETATRON

6.8.3 Working

- Combining Eqs. (6.10), (6.11) and (6.12)

$$\frac{d\,\emptyset}{dt} = 2\,\pi\;r_o{}^2\;\frac{d\,B_0}{dt} \qquad (6.13)$$

- Integrating with respect to t, we get

$$\emptyset = \;2\,\pi\;r_o{}^2\,B_0 \qquad (6.14)$$

6.8.3 Working

This equation must be satisfied during the entire accelerating period, if the electrons have to be the same orbit.

If B′ is the average magnetic field over the whole area of the orbit, then

Total magnetic flux

$$\emptyset = 2\pi\ {r_o}^2 B' \qquad (6.15)$$

6.8 BETATRON

$$\emptyset = 2\pi\ r_o{}^2 B_0 \quad (6.14)$$

6.8.3 Working

B′ is the average magnetic field over the whole area of the orbit, then Total magnetic flux

$$\emptyset = 2\pi\ r_o{}^2 B' \qquad (6.15)$$

Comparing Eqs. (6.14) and (6.15), we get

$$\emptyset = \frac{1}{2} B' \qquad (6.16)$$

This is known as ***Betatron condition***.

6.8 BETATRON

6.8.4 Average Energy per Orbit

We assume that the total flux ∅ varies as

$$\emptyset_0 = \sin \omega\, t$$

so the average kinetic energy gained per orbit.

E′ in the first quarter of the cycle, i.e.

$$\frac{T}{4} = \frac{\pi}{2\,\omega}$$

6.8.4 Average Energy per Orbit

E′ in the first quarter of the cycle is given by the integral of the product of electron charge and induced e.m.f. over this time span.

Mathematically, it is written as

$$E' = \frac{e\emptyset_0}{\frac{\pi}{2\omega}} \int_0^{\pi/2} \frac{d}{dt} \sin(\omega t)\, dt \qquad (\ 6.17)$$

6.8 BETATRON

6.8.4 Average Energy per Orbit

$$E' = \frac{e\emptyset_0}{\frac{\pi}{2\omega}} \int_0^{\pi/2} \frac{d}{dt} \sin(\omega t)\, dt \qquad (6.17)$$

Or

$$E' = \frac{2\, e\, \omega\, \emptyset_0}{\pi}$$

but $\emptyset_0 = 2\ \pi\, r_o{}^2\, B_0$ So, the average energy gained in one orbit is

$$E' = \frac{2\, e\omega\, [2\ \pi\, r_o{}^2\, B_0]}{\pi} \Rightarrow \boxed{E' = 4\, e\, \omega\, r_o{}^2\, B_0} \qquad (6.18)$$

6.8 BETATRON

6.8.5 Calculation of Final Energy of Electrons

Distance travelled by the electron in T/4 seconds

$$= \text{Velocity x time} = \frac{v\,\pi}{2\,\omega} \qquad (6.19)$$

where *v* is the velocity of the electron, which is fairly close to velocity of light.

6.8.5 Calculation of Final Energy of Electrons

Distance travelled by the electron in one orbit = $2\ \pi\ r_o$

Number of orbits that electron makes

$$= \frac{Total\ distance\ travelled\ by\ the\ electron}{distance\ travelled\ by\ the\ electron\ in\ one\ orbit}$$

Number of orbits $= \dfrac{v}{2\ \omega r_0}$ (6.20)

6.8 BETATRON

6.8.5 Calculation of Final Energy of Electrons

Combining Equations (6.18) and (6.19), we get

$$E' = 4\, e\, \omega\, r_o{}^2\, B_0 \qquad (6.18)$$

Distance travelled by the electron in T/4 seconds = Velocity x time $= \frac{v\, \pi}{2\, \omega}$ (6.19)

Total energy $= v\, e\, r_o\, B_0$ (6.21)

6.8 BETATRON

Example:

As an illustration. we will discuss how betatron produces electrons with high energy.

Suppose electron with energy 70 keV are introduced in the doughnut.

The corresponding speed of the electrons comes out to be ≈ 2×10^{10} cm/s.

6.8 BETATRON

Example:

The radius of the orbit is 50 cm and if the electromagnet is powered by an ac frequency of 60 Hz and magnetic field at the orbit is 1 T, let us calculate:

1. Total distance travelled in T/4 seconds.
2. Number of orbits.
3 Average energy per orbit.
4 Total energy.

6.8 BETATRON

1 Total distance travelled in T/4 seconds:

Using Eq. (6.19)

Distance travelled by the electron in T/4 seconds = $\frac{v\,\pi}{2\,\omega}$ (6.19)

$$\text{Distance} = \frac{2 \times 10^{10}\,\pi}{2 \times 2\,\pi \times v}$$

$$= \frac{2 \times 10^{10}}{2 \times 2 \times 60}$$

$$= 8.33\ 10^{7}\ cm$$

$$= 8.33\ 10^{5}\ m$$

6.8 BETATRON

2 Number of orbits: Using Eq. (6.20)

$$\text{Number of orbits} = \frac{v}{2\,\omega r_0}$$

$$\text{Number of orbits} = \frac{2 \times 10^8}{4 \times [2\,\pi \times v] \times r_0}$$

$$= \frac{2 \times 10^8}{4 \times 2\,\pi \times 60 \times 0.5}$$

$$= 2.65 \text{ x } 10^5 \text{ orbits}$$

6.8 BETATRON

3 Average energy per orbit:

Using Eq. (6.18)

$$E' = 4\, e\, \omega\, {r_o}^2 B_0$$

Average energy per orbit = 4 x [16 x 10^{-19}]x [2 x π x v] x 0.5^2 x 1

= 4 x 16 x 10^{-19} x 2 x π x 60 x 0.5^2 x 1

= 6.03 x 10^{-17} J

= 375 eV

6.8 BETATRON

4 Total energy: Using Eq. (6.21)

Total energy $= v\, e\, r_o\, B_0$

$=[2 \times 10^{8}] \times [1.6 \times 10^{-19}] \times [0.5] \times [1]$

$= 1.6 \times 10^{-11}$ J

$= 100$ MeV

6.9 SYNCHROCYCLOTRONS
OR
FREQUENCY MODULATED CYCLOTRONS

SYNCHROCYCLOTRONS:

- It is a cyclotron with the accelerating supply frequency decreasing as the particles become relativistic and begin to lag behind.
- Although in principle they can be sealed up to any energy they are not built any more as the synchrotron is a more versatile machine at high energies

6.9.1 Principle

- Synchrocyclotron is based on the principle that loss of resonance at high velocities, where there is an appreciable increase in the mass of the particle due to relativistic effects can be compensated by decreasing the applied ac oscillating frequency.

6.9.1 Principle

- Synchrocyclotron is based on the principle that loss of resonance at high velocities, where there is an appreciable increase in the mass of the particle due to relativistic effects can be compensated by decreasing the applied ac oscillating frequency.

6.9.2 Construction

- The basic design of a synchrocyclotron is similar to that of a cyclotron.
- The first synchrocyclotron was built at Berkeley, USA.
- It had magnets with pole pieces diameter of **184 inches** or about **470 cm**.
- It used a **single dee**, instead of two dees as is in the conventional cyclotron.

6.9.2 Construction

- It is worthwhile to mention here that the first two cyclotrons: a 4 inch and other 11 inch, constructed in early development of the cyclotron principle has only a single dee.
- As a general rule, one dee electrode is adequate when the applied **ac potential is not too high** the other terminal of the ac oscillator is then grounded.

6.9.2 Construction

- In the 184-inch synchrocyclotron frequency of the oscillator was varied from **36 MHz** to **18 MHz**.
- **Thirty-six MHz** was the initial frequency when the protons were moving with non relativistic velocity and as they gained energy, the frequency was slowly reduced to **18 MHz**, when protons gained full energy.

6.9.2 Construction

- This change in the frequency was done about **64 times per second**. The magnetic field of the magnets was about **2.3 tesla**. This accelerator was capable of accelerating protons to **740 MeV**.
- However, if we compare the output of cyclotron with that of synchrocyclotron, there is a difference.

6.9.2 Construction

- In a cyclotron, the flow of accelerated ions is regarded as continuous, although it actually consists of a series of pulses corresponding to each cycle of the oscillating potential.
- For a frequency of **20 MHz**, there would be **20 million pulses** of ions that reach the target per second.

6.9.2 Construction

- In the synchrocyclotron, however, a pulse of ions is carried from the ion source at the centre to the periphery of the dee as the frequency of the oscillating potential is decreased from its initial value (**36 MHz**) to the final value (**18 MHz**).

6.9.2 Construction

- The frequency then returns to its original high value (**36 MHz**) and another pulse of the ions is carried from the ion source to the periphery and so on. The rate at which ion pulses are produced depends on the repetition rate of the frequency.

6.9.3 Theory

The frequency of revolution or angular frequency of the ions is

$$\omega_c = \frac{B\,q}{m} = \frac{B\,q\,c^2}{m\,c^2}$$

where m c^2 is the total energy of the ions which includes the kinetic energy T and the rest mass energy $m_0\,c^2$

$$mc^2 = T + m_0\,c^2$$

6.9.3 Theory

$$\omega_c = \frac{B\,q}{m} = \frac{B\,q\,c^2}{m\,c^2} \Rightarrow \omega_c = \frac{B\,q\,c^2}{T + m_0\,c^2}$$

Now

$$\omega_c = 2\,\pi\,f \;or\; f = \frac{\omega}{2\,\pi}$$

Here *f* is taken as the frequency of ac oscillator as it is in phase with the frequency of revolution of ions.

6.9.3 Theory

$$f = \frac{B\,q\,c^2}{2\,\pi\,(T + m_0\,c^2)} \qquad (6.22)$$

This frequency will have maximum value when T ≈ 0, or

$$f_{max} = \frac{B\,q\,c^2}{2\,\pi\;m_0\,c^2} \qquad (6.23)$$

6.9.3 Theory

and will have minimum value when $T \sim T_{max}$

or

$$f_{min} = \frac{B\, q\, c^2}{2\,\pi\,(T_{max} + m_0\, c^2)}$$

6.9.3 Theory

Solving these two equations for T_{max} we get

$$T_{max} = \frac{f_{max} - f_{min}}{f_{max}\, f_{min}} \frac{B\, q\, c^2}{2\, \pi} \qquad (6.24)$$

6.9.4 Advantages

1. It is capable of accelerating positively charged particles to very high energies.
2. Normally, low power (~15 kW) oscillators are needed in these accelerators as a source of ac potential.
3. With one dee, the electrical and mechanical design become simple.

6.9.5 Limitation

1. The output beam current is very low around microamperes or even lower than that.

Thanks