TYBSc [Semester-6] Physics

US06CPHY23 Nuclear Physics

UNIT- 4 Part 1 Lecture 2

Detectors and Accelerators

Ch 6 Accelerators: Topics

Ch 6 Accelerators: Topics

Recommended Books:

Nuclear and Particle Physics (2nd edition)
V K Mittal, R C Verma and S C Gupta
PHI Learning Pvt. Ltd.

6.5 Linear Accelerator
or
Drift Tube accelerator

6.5 LINEAR ACCELERATOR (LINAC)

LINEAR ACCELERATOR (LINAC)

- It is an electrical device for the **acceleration of subatomic particles like** electron, proton, ion etc.
- The linear particle accelerator (LINAC) is also called **DRIFT TUBE ACCELERATOR.**

LINEAR ACCELERATOR (LINAC)

- It has **many applications**
  - **To generate X-rays in hospitals.**
  - To an injector into higher energy accelerators at a dedicated experimental **particle physics laboratory.**

6.5 LINEAR ACCELERATOR (LINAC)

LINEAR ACCELERATOR (LINAC)

- The **design** of a LINAC depends on the **type of particle** that is being accelerated electron, proton or ion.

- They **range in size** from a cathode-ray tube to the 2-mile-long Stanford Linear Accelerator Center in California.

LINEAR ACCELERATOR (LINAC)

- **High power LINACS** are also being developed for production of electrons at relativistic speeds, required since fast electrons travelling in an arc will lose energy through synchrotron radiation, this limits the maximum power that can be imparted to electrons in a synchrotron of a given size.

6.5 LINEAR ACCELERATOR (LINAC)

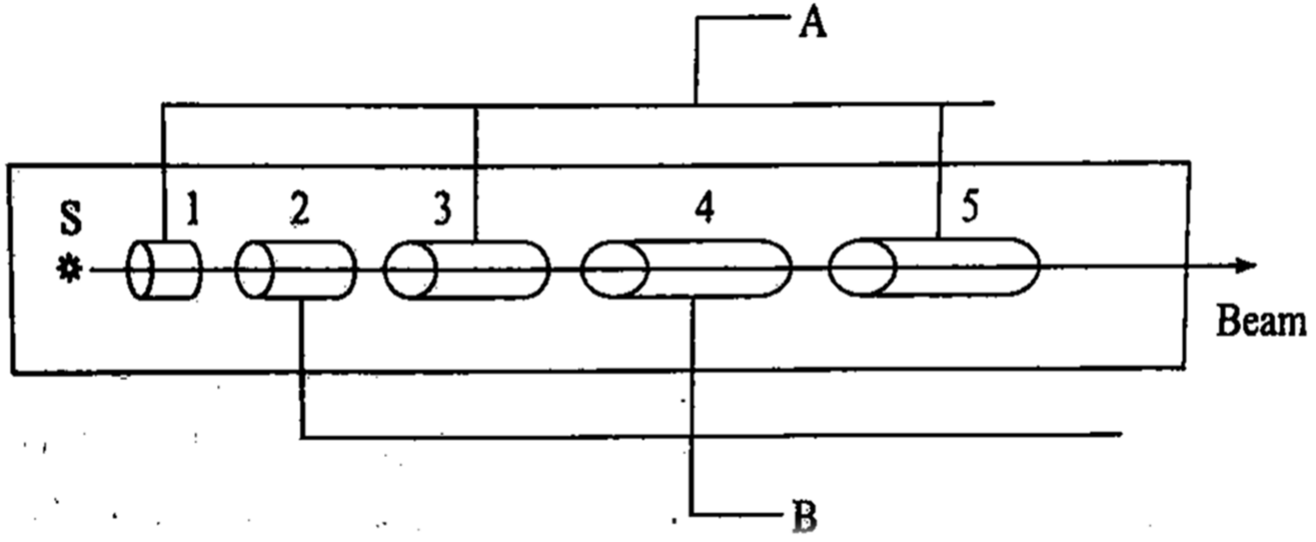
6.5.1 Principle

If a charged particle passes through **cylindrical electrodes** joined in **series** and a **high frequency oscillating field** is applied to these electrodes in such a way that alternating electrodes are positive and negative, the particle experiences acceleration when it passes through the gap between electrodes

6.5 LINEAR ACCELERATOR (LINAC)

6.5.2 Construction

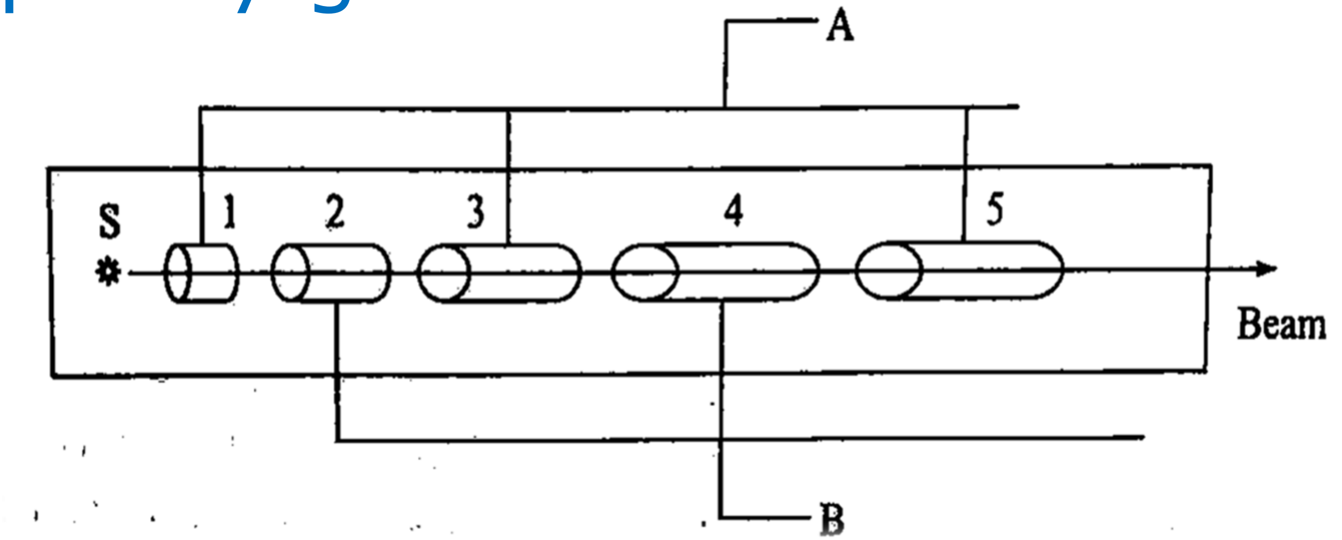
Linear accelerator consists of a number of hollow cylindrical electrodes of increasing length, arranged in a straight line as shown in Figure.



6.5 LINEAR ACCELERATOR (LINAC)

6.5.2 Construction

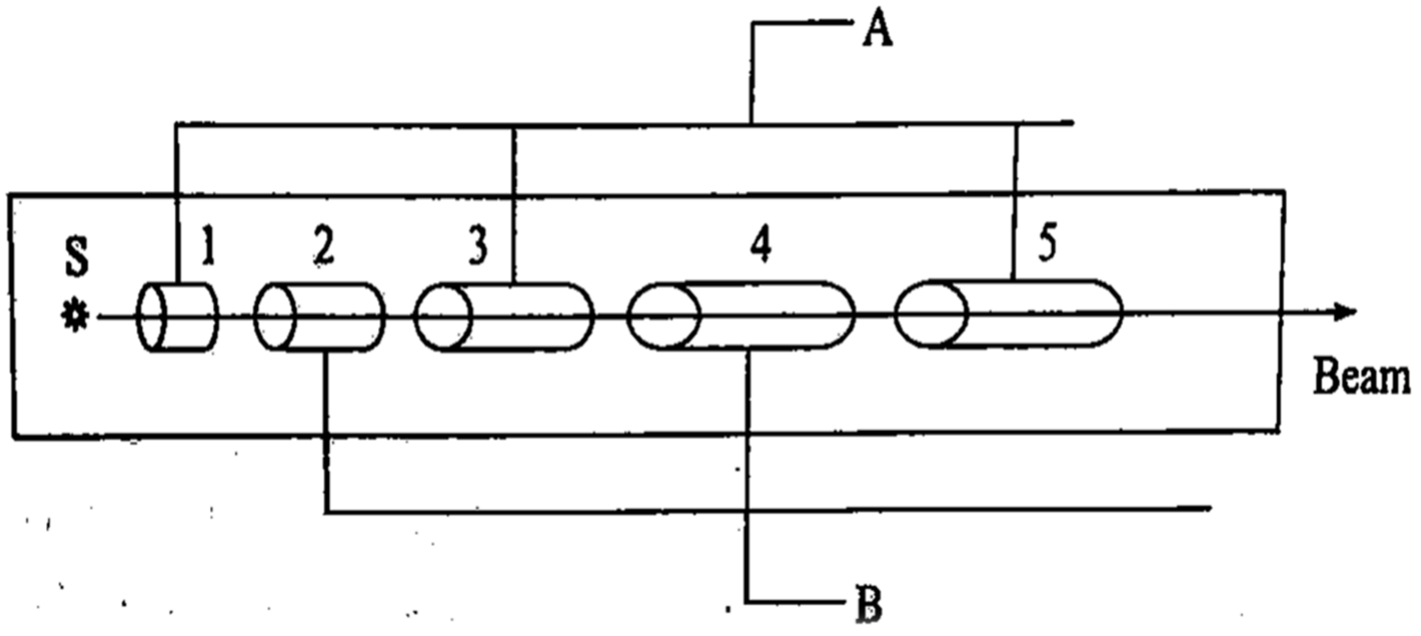
- **Alternate electrodes**, i.e., the first, third, fifth etc. are joined to one terminal **A** and the second, fourth, sixth, etc. to **other terminal B** of a high frequency generator.



6.5 LINEAR ACCELERATOR (LINAC)

6.5.2 Construction

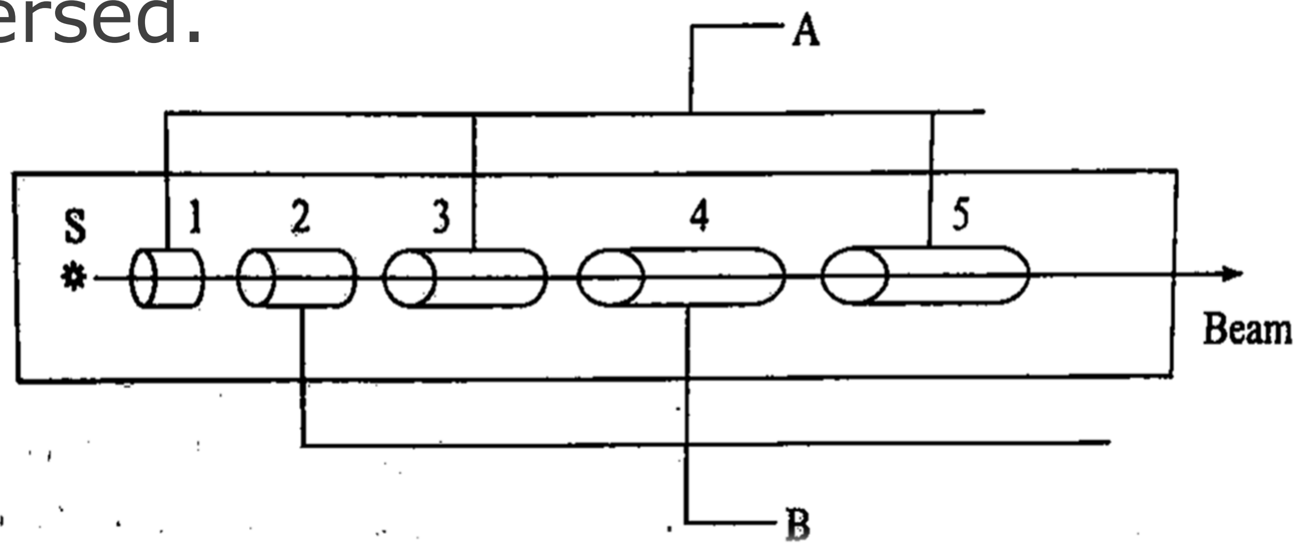
- In this arrangement alternate electrodes carry **opposite electrical potentials**.



6.5 LINEAR ACCELERATOR (LINAC)

6.5.2 Construction

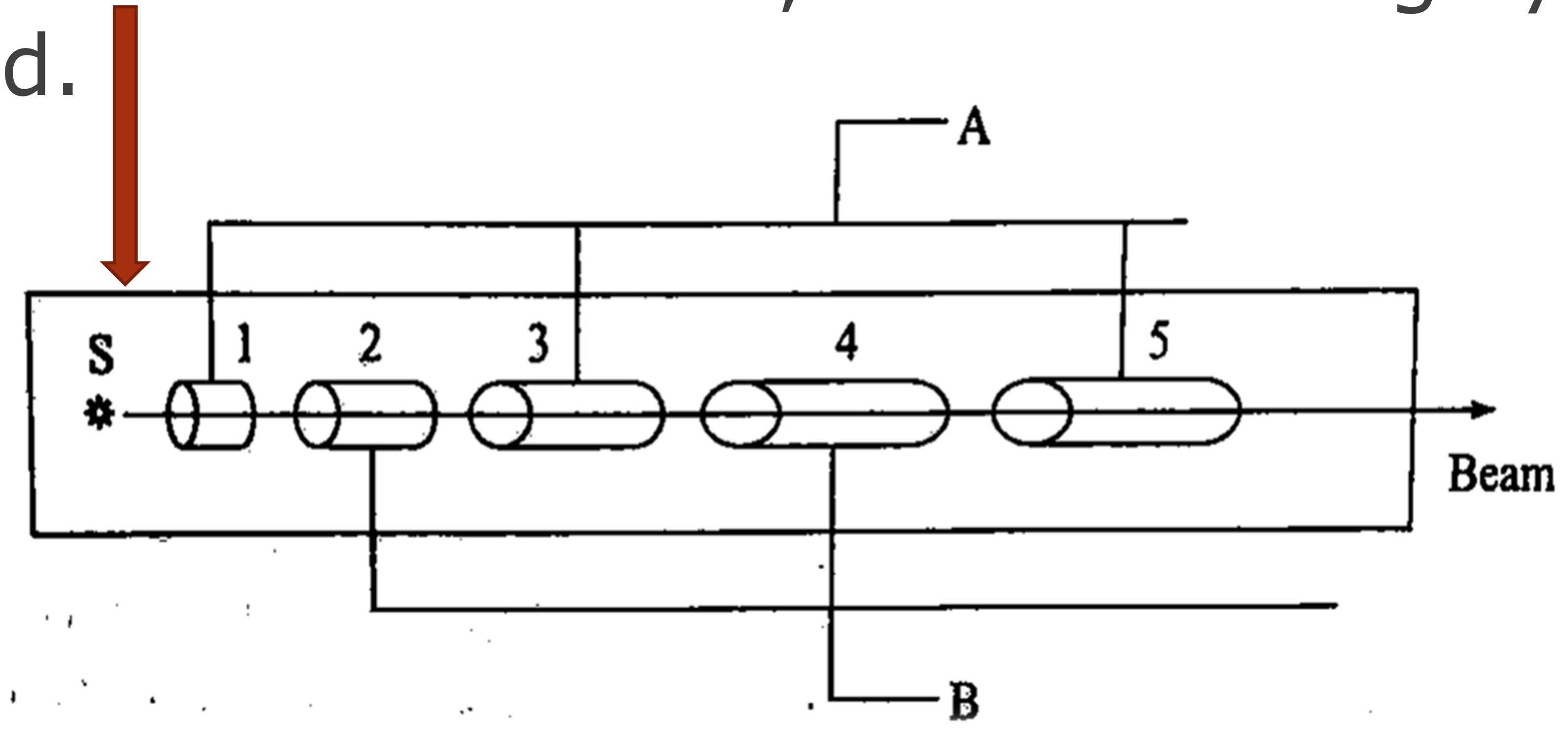
- For example, in a particular **half-cycle** of the oscillation all the **odd numbered electrodes** would be **positive** whereas those with **even numbers** would be **negative**. In the next half-cycle, the potentials are reversed.



6.5 LINEAR ACCELERATOR (LINAC)

6.5.2 Construction

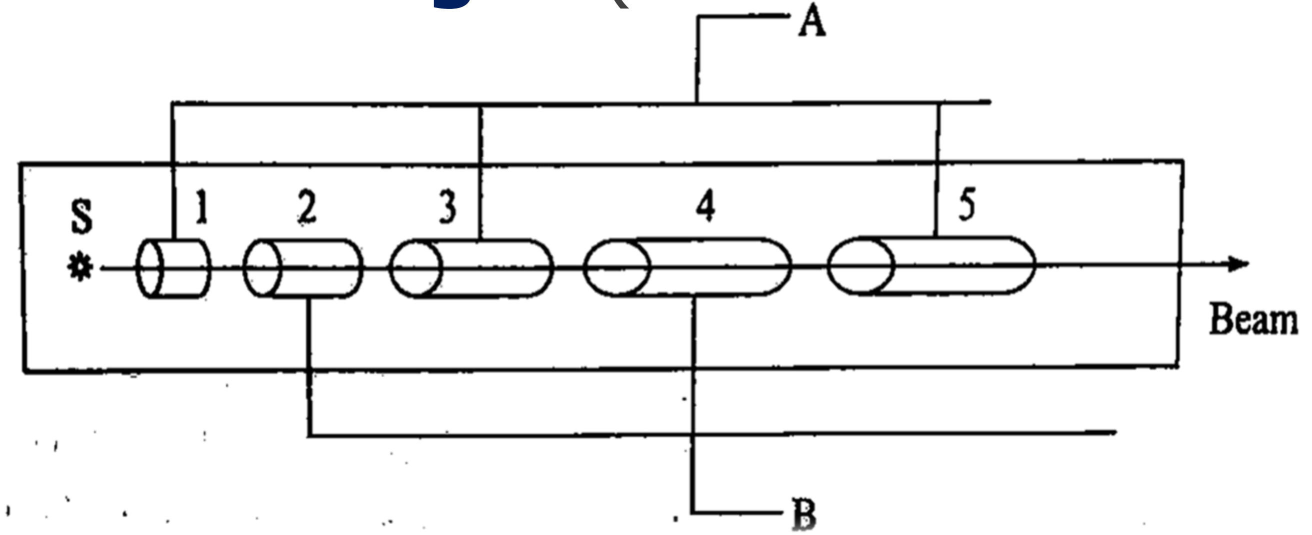
- This whole arrangement of electrodes is enclosed in another tube, which is highly evacuated.



6.5 LINEAR ACCELERATOR (LINAC)

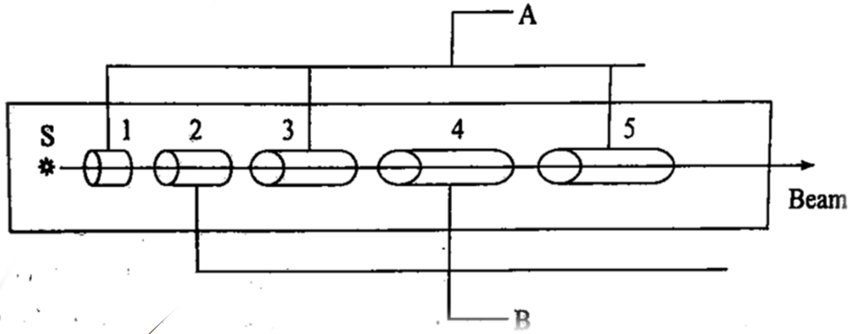
6.5.2 Construction

- **S** is an **ion source** placed one end of the accelerator, where the length of the electrodes is shortest.
- It inject **positive ions** in the accelerator and on the opposite side, there is a **target** (not shown in the diagram)



6.5 LINEAR ACCELERATOR (LINAC)

6.5.3 Working

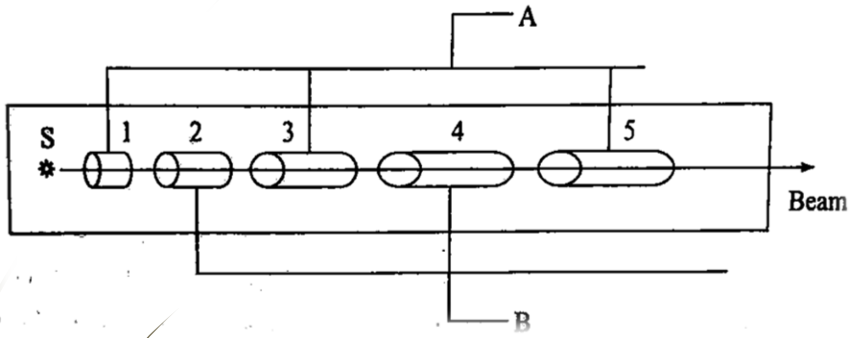


$$L_n = \frac{1}{2f}\sqrt{\frac{2\,n\,q\,V_0}{m}}$$

- Suppose **positively charged ions** from the ion source S move from left to right along the common axis of the cylindrical electrodes.
- While passing through electrode 1, ions receive no acceleration, since they are moving in a field-free region.
- However, in traversing the gap between the 1st and 2nd electrodes, the ions are in the region in which there is a difference of potential.

6.5 LINEAR ACCELERATOR (LINAC)

6.5.3 Working

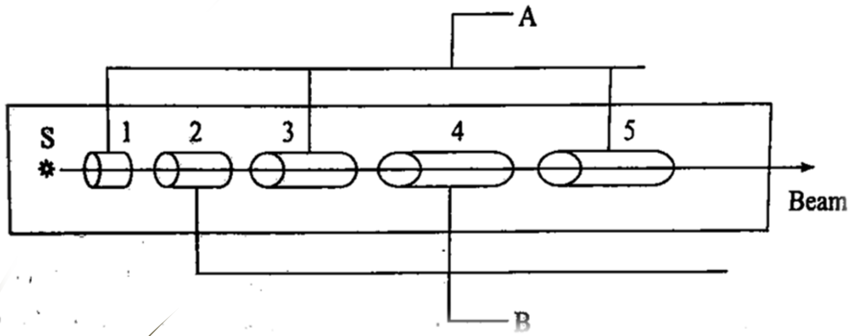


$$L_n = \frac{1}{2\,f}\sqrt{\frac{2\,n\,q\,V_0}{m}}$$

- If the 1st electrode is positive and 2nd is negative, the positively charged ions are accelerated while crossing the gap.
- The positive ions then enter the 2nd electrode, and travel through it at a constant speed but at a higher velocity than in the 1st electrode.

6.5 LINEAR ACCELERATOR (LINAC)

6.5.3 Working

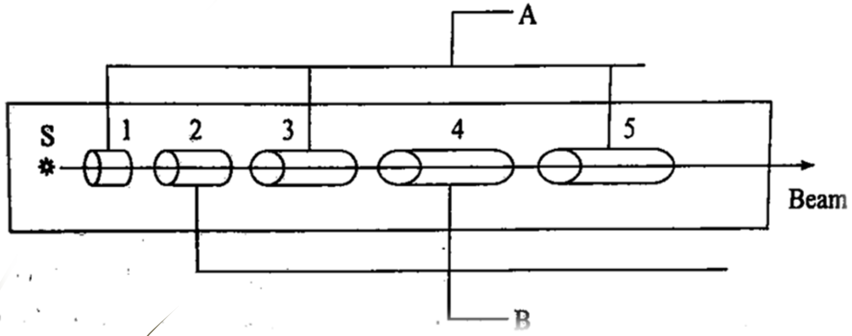


$$L_n = \frac{1}{2f}\sqrt{\frac{2\,n\,q\,V_0}{m}}$$

- The length of this electrode is such that just as the ions reach the gap between 2nd and 3rd electrodes, the polarity of electrodes gets reversed, *i.e.* 2nd electrode becomes positive and 3rd becomes negative. This causes the positive ions to get additional acceleration in this gap also.

6.5 LINEAR ACCELERATOR (LINAC)

6.5.3 Working



$$L_n = \frac{1}{2f}\sqrt{\frac{2\,n\,q\,V_0}{m}}$$

- By making successive electrodes increasingly longer, to compensate for the increasing speed of the positive ions, it is possible to keep the ions in phase with the oscillating frequency.

6.5 LINEAR ACCELERATOR (LINAC)

6.5.3 Working

- The positive ions gain energy each time they go from one electrode to the next.
- If ***q*** is the charge on the ion, ***V***, is the peak value of the ac frequency applied to the electrodes, then in each gap ions acquire energy qV_0 eV.
- If there are n such gaps, then

$$\text{Energy acquired in n gaps} = n\,q\,V_0 \text{ eV.} \qquad (6.1)$$

6.5.3 Working

The **total kinetic energy** acquired in the nth gap

$$= \frac{1}{2}\, m\, v_n^2$$

Equating these two energies

$$\frac{1}{2}\, m\, v_n^2 = n\, q\, V_0$$

Or $$v_n = \sqrt{\frac{2\, n\, q\, V_0}{m}} \qquad (6.2)$$

6.5 LINEAR ACCELERATOR (LINAC)

6.5.3 Working

Or $$v_n = \sqrt{\frac{2\,n\,q\,V_0}{m}} \qquad (6.2)$$

- In these calculations, we have assumed that $v_n << c$.
- Equation (6.2) shows that in order to get a high-energy
- the peak voltage V_0 of the oscillator should be higher.
- the number of gaps **n** should be as large as possible.

6.5 LINEAR ACCELERATOR (LINAC)

6.5.3 Working

$$v_n = \sqrt{\frac{2\,n\,q\,V_0}{m}}$$

- If ***f*** is the frequency of the oscillating potential, the time duration of a half-cycle is $= \frac{1}{2\,f}$
- The time ions take in passing through a nth cylinder of length $L_n = \frac{L_n}{v_n}$
- Where v_n is the velocity of the ions in nth cylinder.

6.5 LINEAR ACCELERATOR (LINAC)

6.5.3 Working

- For synchronization, we must have $\frac{1}{2\,f} = \frac{L_n}{v_n}$
- Or $L_n = \frac{v_n}{2\,f}$
- Substituting for v_n from Eq. (6.2)

$$L_n = \frac{1}{2\,f}\sqrt{\frac{2\,n\,q\,V_0}{m}} \qquad (6.3)$$

6.5 LINEAR ACCELERATOR (LINAC)

- In **1931**, **Sloan and Lawrence** built the **first** linear accelerator and using this accelerator they accelerated **Hg ions** to **2.85 MeV** with peak oscillating voltage of **42 kV** only.

- Linear accelerators generally require very big set-ups. For example, LINAC at **Los Almos in USA** accelerates protons to **800 MeV** is **805 metres long**.

6.5 LINEAR ACCELERATOR (LINAC)

6.5.4 Advantages

1. Requirement of generating very high-voltages-million volts range is avoided in these accelerators.

2. They are **economical** for obtaining very high-energy particle beams.

3. They provide **well-collimated beam** of accelerated ions.

6.5 LINEAR ACCELERATOR (LINAC)

6.5.5 Limitations

1. They are inconveniently **long in size**.

2. They require **extremely high frequency** and **high-voltage oscillator**.

6.7 MAGNETIC RESONANCE ACCELERATORS OR CYCLOTRONS

6.7 CYCLOTRONS

- **MAGNETIC RESONANCE ACCELERATORS OR CYCLOTRONS**

- Drawback of previous accelerators:
  - Van de Graaff or Tandem accelerators.

- **Breakdown of electric insulation and sparking**
- Particles cannot be accelerated to very high energies in electrostatic accelerators.

6.7 CYCLOTRONS

- **MAGNETIC RESONANCE ACCELERATORS OR CYCLOTRONS**

- Drawback of previous accelerators:
  - **linear accelerators – Size**

- Very high energies, the **length of** becomes abnormally long (tens of kilometres).

6.7 CYCLOTRONS

- **CYCLOTRONS**

- In 1930, **Lawrence and Livingston** suggested an **alternative way** of accelerating particles.

- Use of a magnetic field to make the charged particles follow a spiral path of increasing radius.

6.7 CYCLOTRONS

CYCLOTRONS

- A beam of charged particles makes perhaps hundreds of revolutions, through the device and in each revolution, particles receive one or two small energy increments until the particle energy reaches the desired value.
- This type of accelerators were initially called **magnetic resonance accelerators** but later named as **Cyclotrons**.

6.7 CYCLOTRONS

CYCLOTRONS

- In 1930, **Lawrence and Livingston** built a first experimental cyclotron, which was able to accelerate protons to **80 keV** using an oscillator of peak voltage of only **2000 volts**.

- The diameter of magnet pole pieces was **4 inches.** Later on in 1932, another cyclotron was built which could accelerate protons to **1.2 MeV**.

- It had a magnet with pole faces **11 inches** in diameter.

CYCLOTRONS

- The diameter of the pole faces of the cyclotron is generally used to describe the **size of the cyclotron**. For example, if the diameter of poles of a magnet of a cyclotron is **88 inches**, it would be referred as an "**88-inch cyclotron**".

- **Larger the diameter** of the pole face of the cyclotron magnet, **greater** would be the **energy** of the accelerated particles.

6.7 CYCLOTRONS

6.7.1 Principle

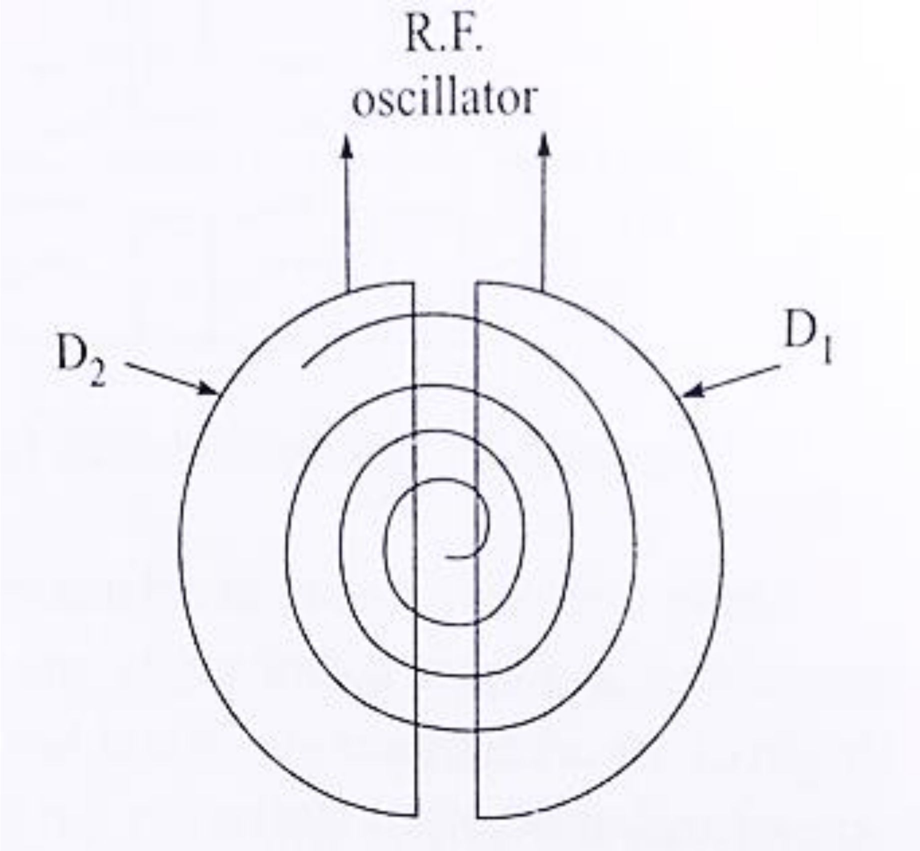
Cyclotron is based on the principle that charged particles acquire energy when they repeatedly move through an alternating electric field along a spiral path.

They move in a spiral path by a perpendicular magnetic field.

6.7 CYCLOTRONS

6.7.2 Construction

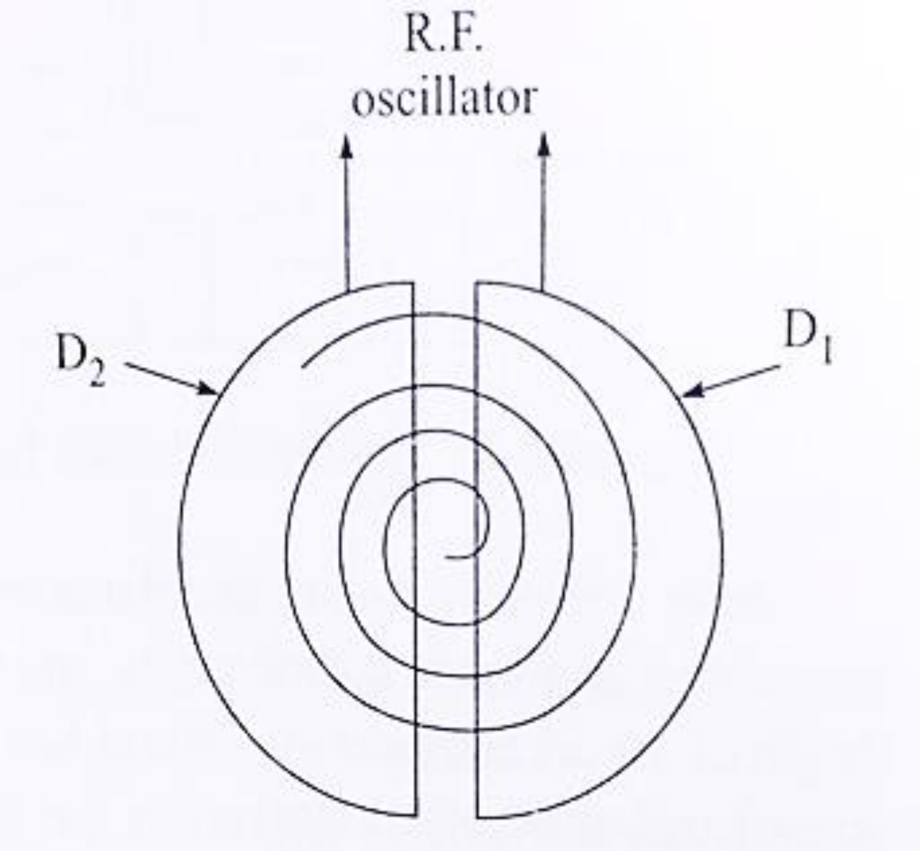
- It consists of **two D-shaped hollow semicircular chambers** D_1 and D_2 called "**Dees**" because of their shapes.



6.7 CYCLOTRONS

6.7.2 Construction

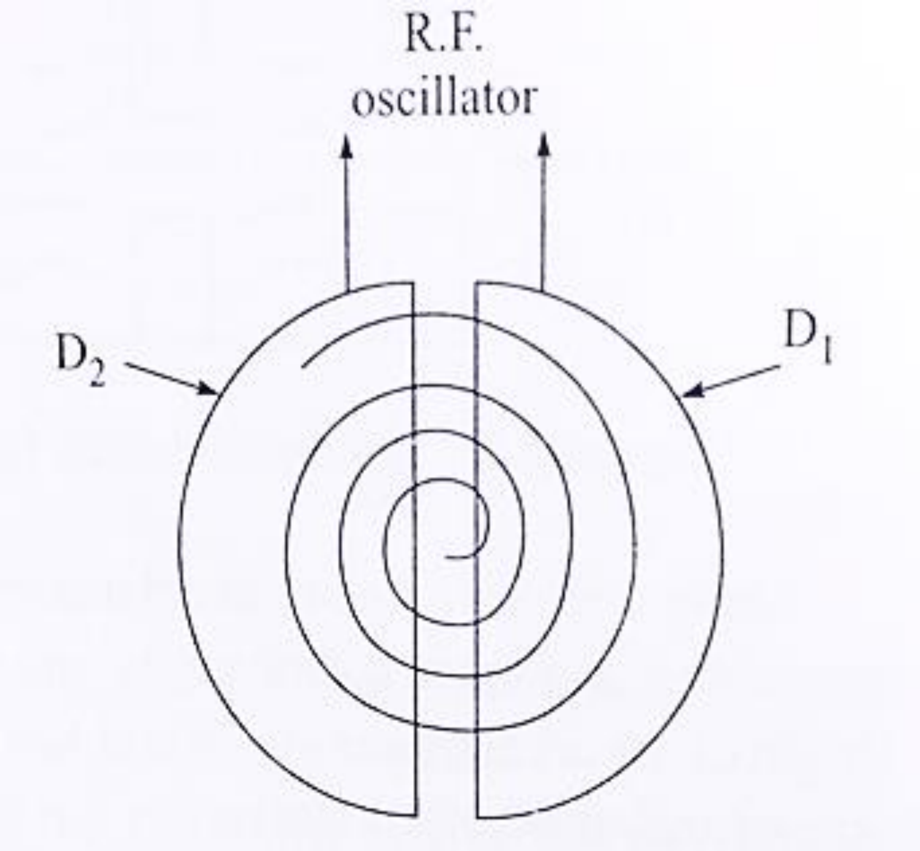
- These dees are placed with their diametric edges parallel to each other and are slightly separated from each other as shown in Figure.



6.7 CYCLOTRONS

6.7.2 Construction

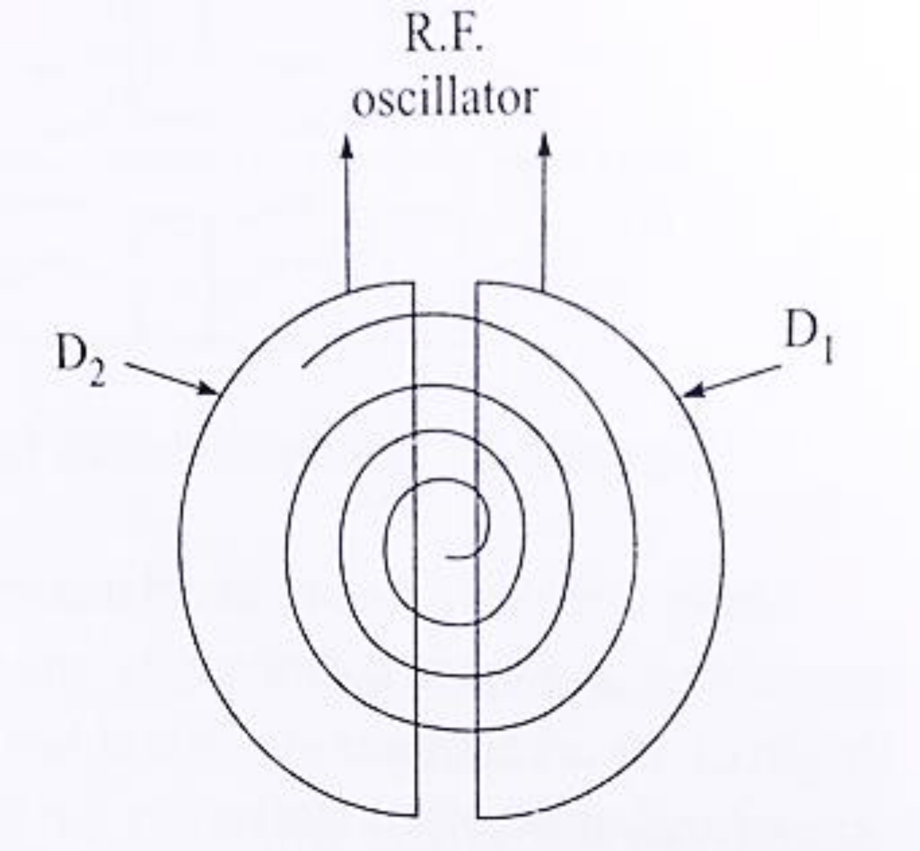
- These dees are connected to the terminals of an alternating high frequency (**10-20 MHz**) and high voltage (**10-50 kV**) peak value.



6.7 CYCLOTRONS

6.7.2 Construction

- This arrangement makes one dee **positive** and other **negative** during one **half-cycle** and vice versa in the next half-cycle.



6.7 CYCLOTRONS

6.7.2 Construction

- An **ion source** S of positive ions, such as protons, deuterons, α-particles, etc. is placed in the central region of the gap between the dees D_1 and D_2.
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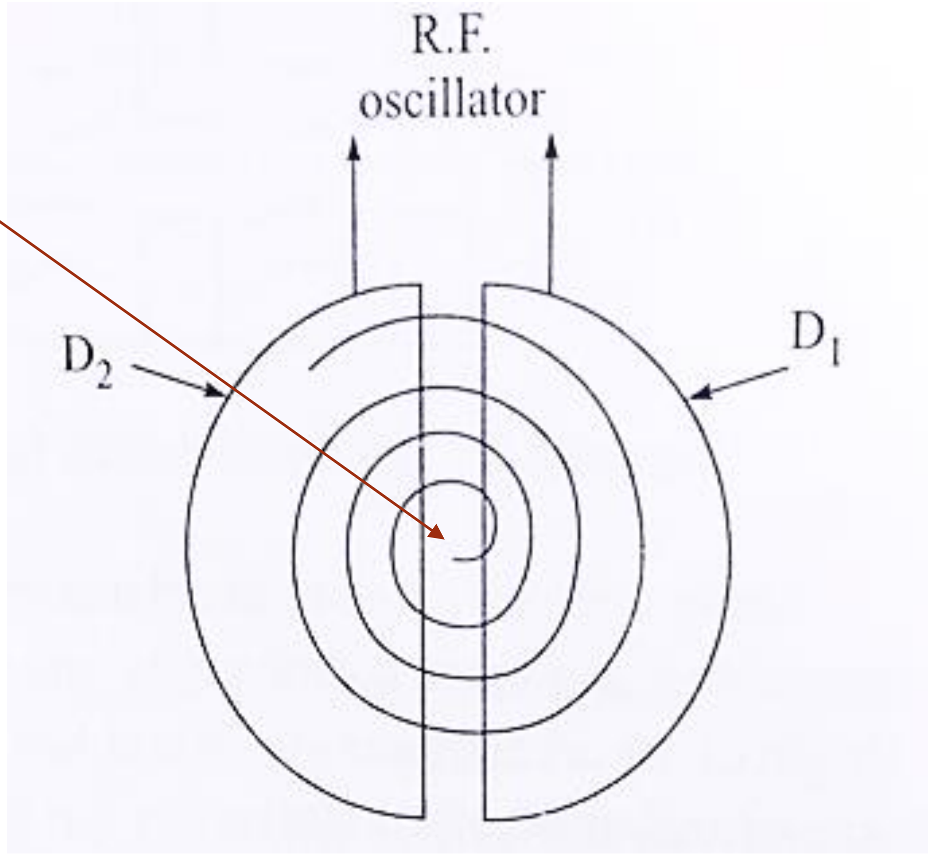
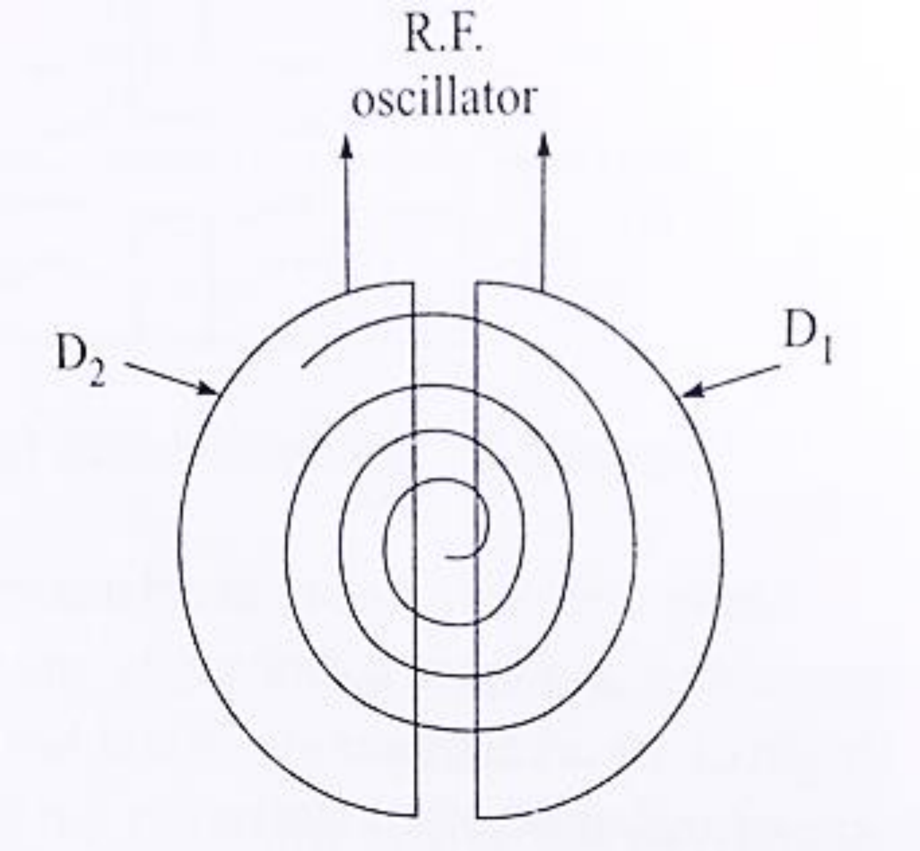


6.7 CYCLOTRONS

6.7.2 Construction

- This entire set-up, *i.e.* dees, ion source, etc. are placed in a **highly evacuated** chamber.



6.7 CYCLOTRONS

6.7.2 Construction

- A **uniform magnetic field** is applied perpendicular to the cross-sectional area of the dees by placing them between the pole faces of a **large electromagnet** as shown in Figure 6.7.
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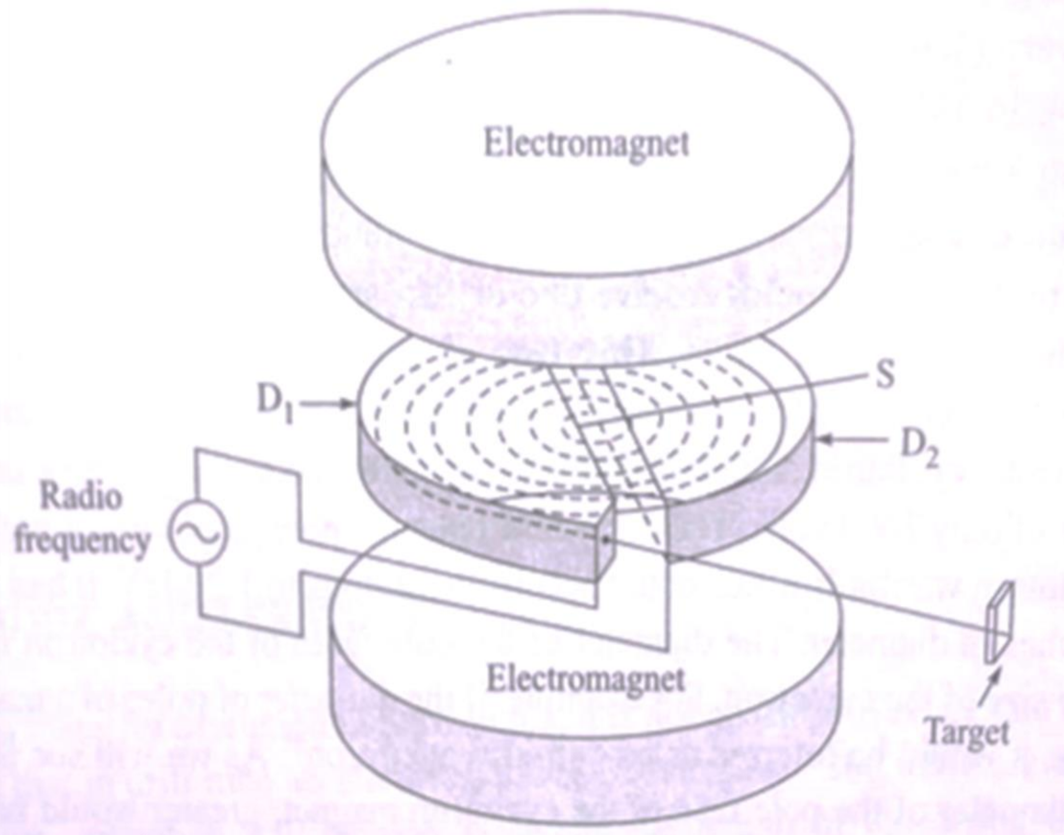
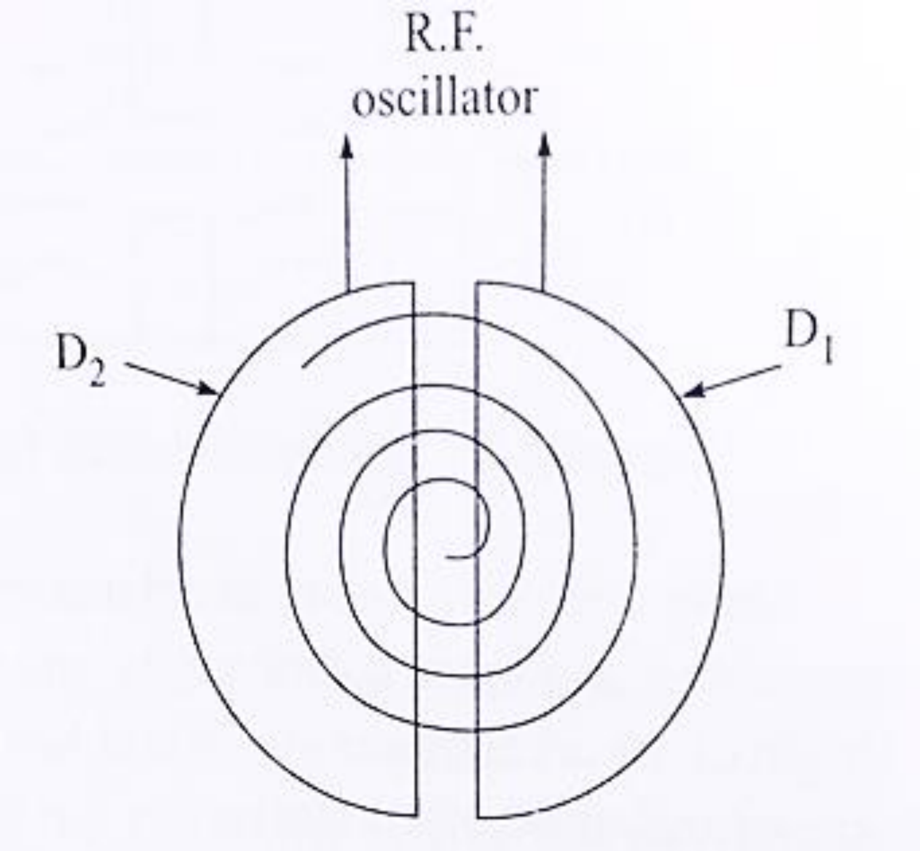


Figure 6.7 Detailed diagram of cyclotron.

6.7 CYCLOTRONS

6.7.3 Working

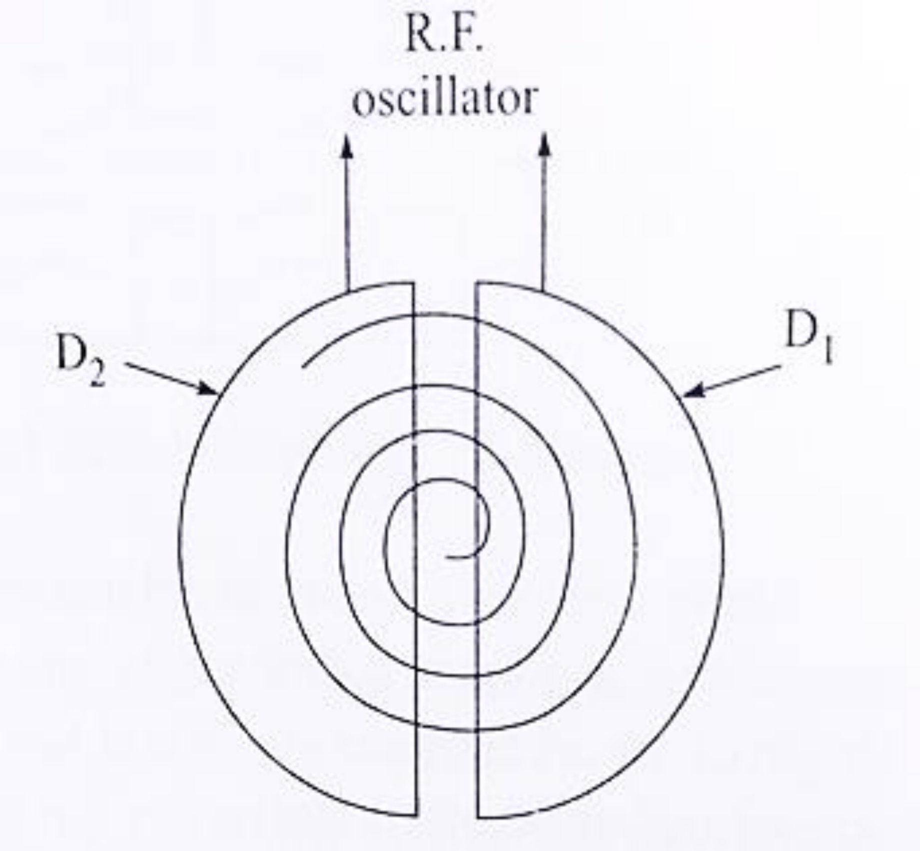
- Ion source S produces positive ions within the gap between dees D_1 and D_2.
- At this instance,
  - **D_1** is **positive** and
  - **D_2** is **negative**.
- The positive ions will be accelerated towards dee D_2 and it enters the dee D_2.



6.7 CYCLOTRONS

6.7.3 Working

- As there is no electric field **inside the dees**, the ions are subjected to perpendicular magnetic field.
- The perpendicular magnetic field acts on the ions and the positive ions move in a **circular path**.

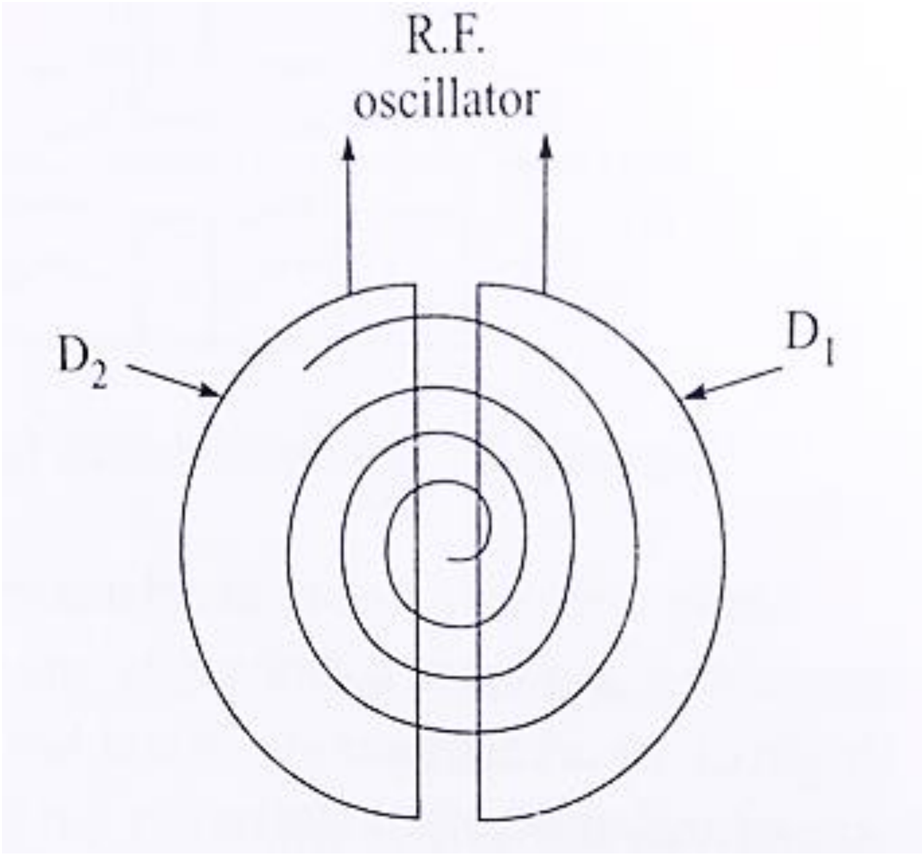


6.7 CYCLOTRONS

6.7.3 Working

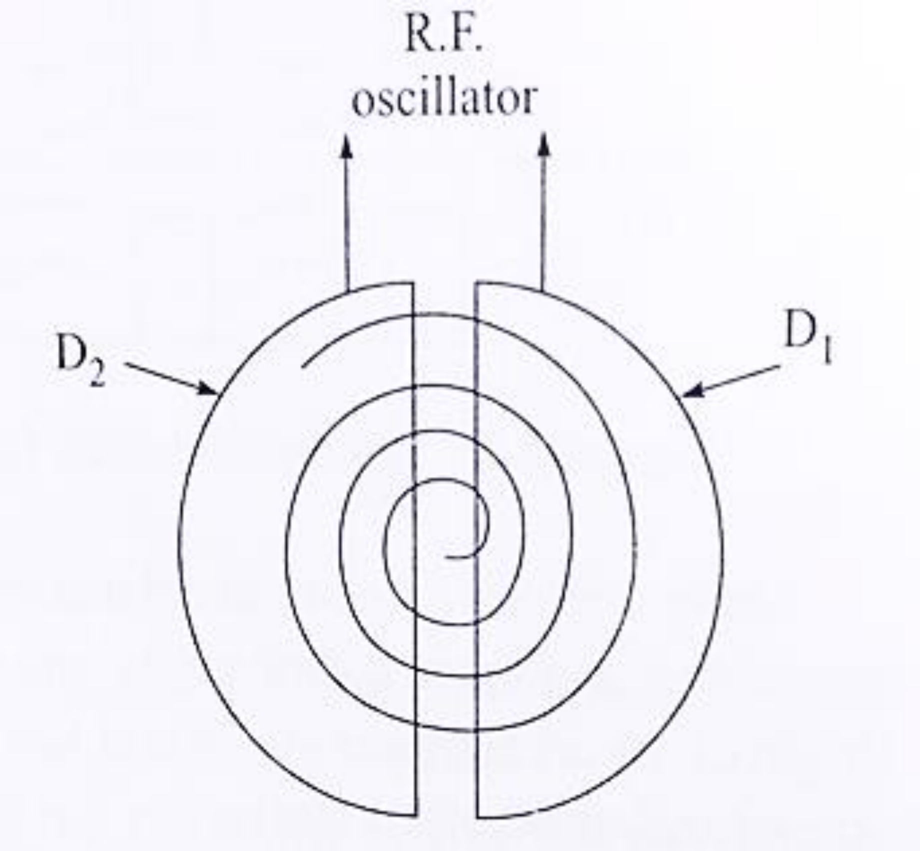
- Once **inside the dee**, they experience **no acceleration** and move with a constant velocity.

- After traversing the semicircular path inside the dee D_2, they return to the gap between the dees.



6.7.3 Working

- The **frequency of the oscillator** is adjusted in such a way that when ions reach the gap, the dee D_2 becomes positive and D_1 becomes negative.

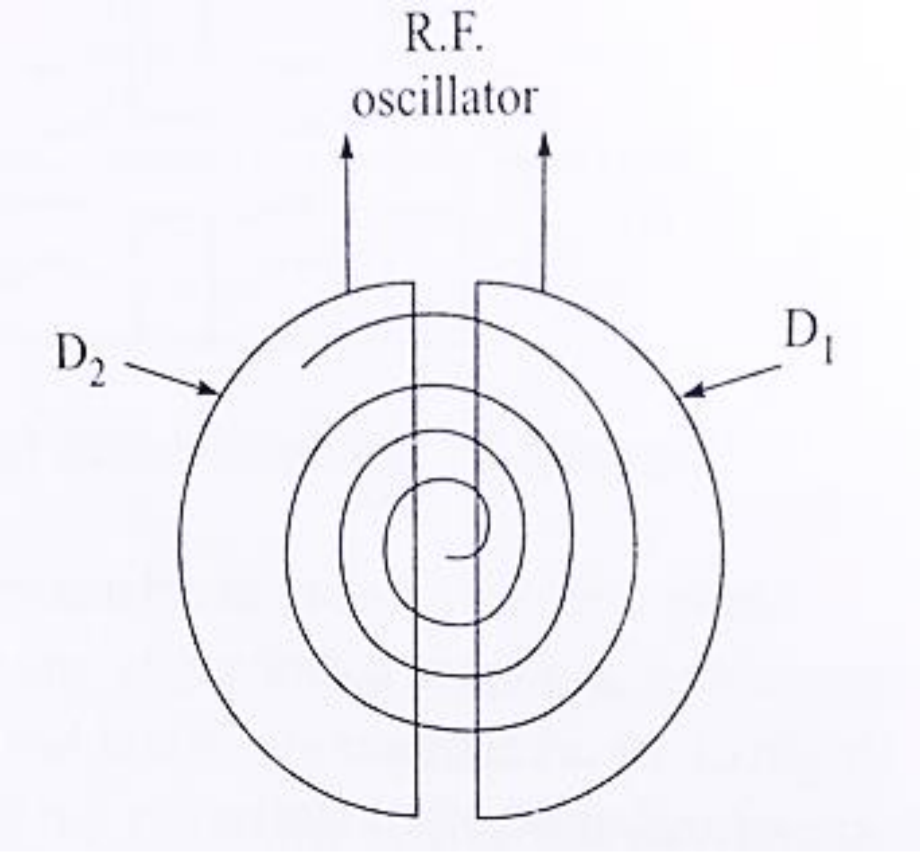


6.7 CYCLOTRONS

6.7.3 Working

- Now the positive ions are accelerated towards dees D_1 thus gaining in kinetic energy.

- The ions enter the dee D_1 with higher kinetic energy compared to the value when it was in dee in the dee D_2, they move again in n circular path with larger radius but with constant velocity.

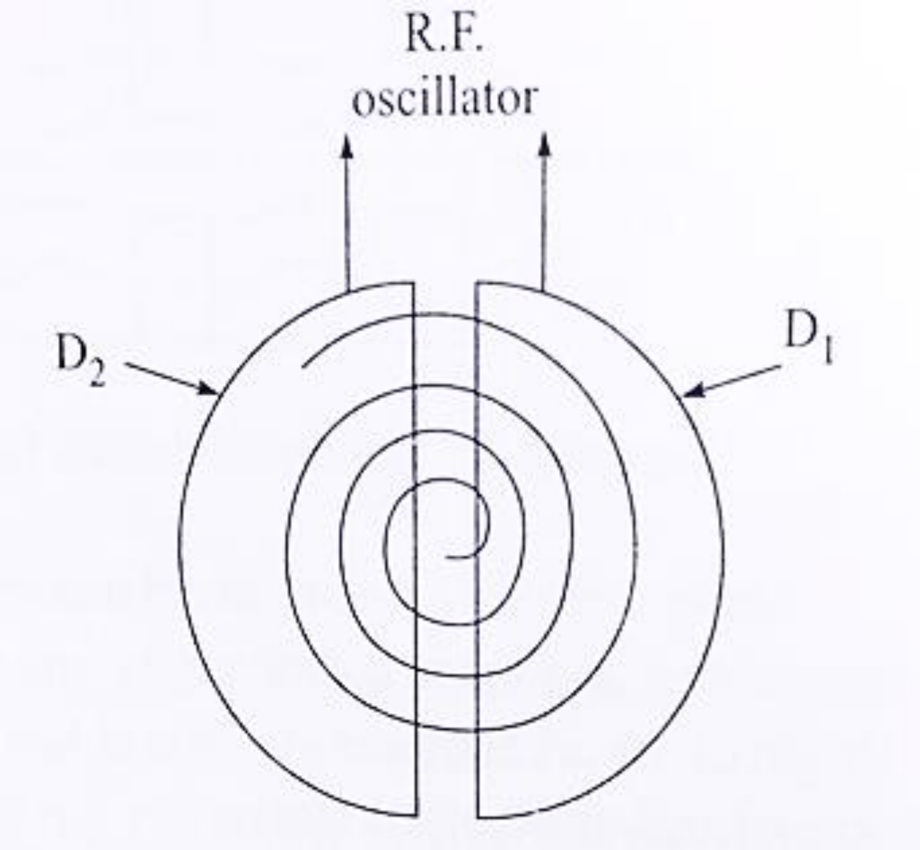


6.7 CYCLOTRONS

6.7.3 Working

- After covering the semicircle inside the dee D_1, the ions reach the gap between the dees.

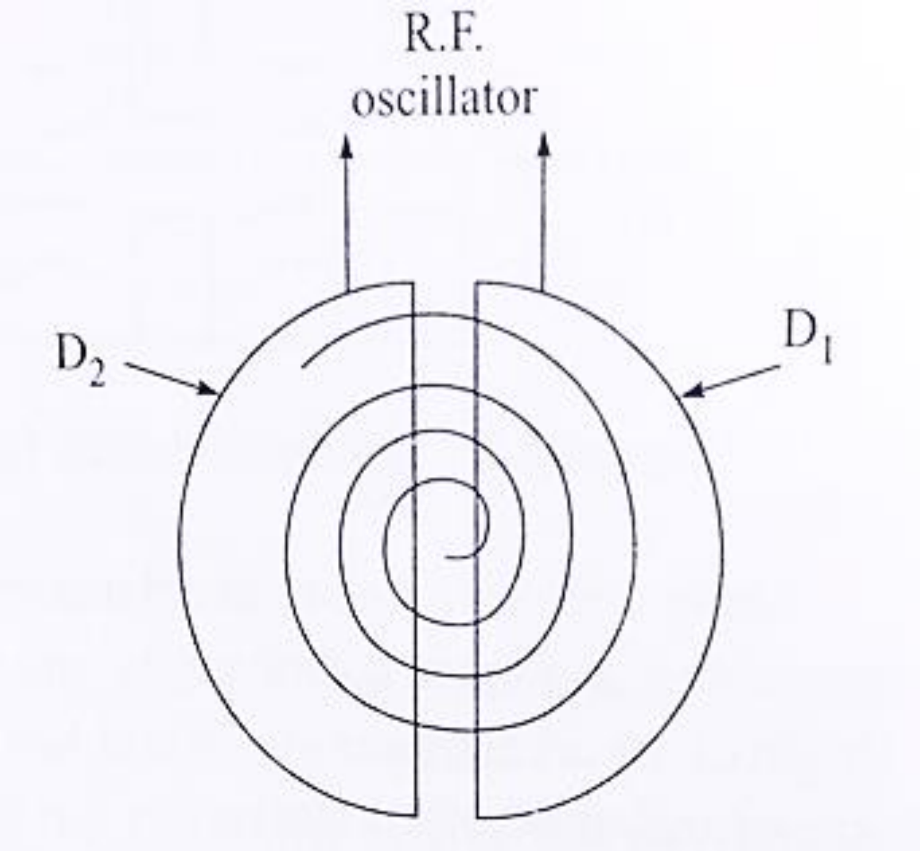
- At this instance again the polarity of the dees is reversed, positive ions again experience acceleration and this process is repeated many times (perhaps hundreds of times).



6.7 CYCLOTRONS

6.7.3 Working

- Finally, when the positive ions reach the **periphery of the dees** after gaining a maximum energy, it is extracted out of the dees by means of high-voltage deflecting plates.



6.7 CYCLOTRONS

6.7.4 Theory

- Let m be the mass of the positive ions to be accelerated, is the charge on the ion.
- The ion moves in a semicircle of radius r with velocity v in a perpendicular magnetic field B

Magnetic force acting on the ion = $q\, v\, B$

6.7.4 Theory

- This force is balanced by the centripetal force $\frac{mv^2}{r}$ acting on the ion. Therefore,

$$\frac{mv^2}{r} = q\,v\,B$$

Or

$$r = \frac{m\,v}{q\,B} \qquad (6.4)$$

6.7.4 Theory

- This force is balanced by the centripetal force $\frac{mv^2}{r}$ acting on the ion. Therefore,

$$\frac{mv^2}{r} = q\,v\,B$$

Or

$$r = \frac{m\,v}{q\,B} \qquad (6.4)$$

- According to this equation, if *m*, *q* and B remain constant, then $r \,\alpha\, v$

6.7 CYCLOTRONS

6.7.4 Theory

- As positive ions gain energy, *i.e.* v increases, the ions move in a semicircle of larger and larger radius.
- Therefore, the path of the ions is spiral in the dees.
- If t is the time taken by the ions to complete a semicircular path, then

$$t = \frac{1}{2}\frac{2\pi}{\omega} = \frac{\pi}{\omega}$$

- where ω is the angular frequency and $\omega = \frac{v}{r}$.

$$r = \frac{m\,v}{q\,B}$$

6.7 CYCLOTRONS

6.7.4 Theory

- Therefore,

$$t = \frac{\pi\,r}{\omega}$$

- Substituting *r* from Eq. (6.4)

$$t = \frac{\pi\,m}{B\,q} \qquad (6.5)$$

- Time T to completing a full circular path is given by

$$T = 2t = \frac{2\,\pi\,m}{B\,q}$$

6.7 CYCLOTRONS

$$T = 2t = \frac{2\pi m}{B q}$$

6.7.4 Theory

- Again, if m, q and are constant, then the time taken by the ion to complete the semicircular path inside the dee is independent of the **velocity of the ion**, **radius of the semicircular path** and **radius of dees of the size of pole pieces of the magnet**.

- Hence, the frequency of the oscillations required to keep the ion in phase is given by

$$f = \frac{1}{T} = \frac{B q}{2\pi m} \qquad (6.6)$$

6.7 CYCLOTRONS

$$r = \frac{m\,v}{q\,B}$$

6.7.4 Theory

If $R = r_{max}$ is the radius of the last orbit from which the ions are extracted out of the dees and $\boldsymbol{V_{max}}$ is the velocity of the ions in the last orbit, then from Eq. (6.4)

$$R = \frac{m\,v_{max}}{q\,B}$$

or

$$v_{max} = \frac{q\,B\,R}{m} \qquad (6.7)$$

6.7.4 Theory

Correspondingly, the maximum kinetic energy of the ions is

$$E_{max} = \frac{1}{2} m v_{max}{}^{2} = \frac{1}{2} m \left(\frac{q\, B\, R}{m} \right)^{2}$$

$$= \frac{1}{2} m \; \frac{q^2\, B^2\, R^2}{m^2} = \frac{1}{2} \; \frac{q^2\, B^2\, R^2}{m} \qquad (6.8)$$

6.7 CYCLOTRONS

6.7.4 Theory

The maximum kinetic energy acquired by the ions can also be calculated by multiplying the charge on the ion ***q***, the peak value of the ac voltage **V** and the total number of circles N, completed by the ion before it is extracted out of the dees.

Therefore,

$$E = N\,q\,V$$

6.7 CYCLOTRONS

6.7.4 Theory

Substituting E_{max} from Eq (6.8)

$$\frac{1}{2}\,\frac{q^2\,B^2\,R^2}{m} = N\,q\,V$$

$$R = \frac{1}{B}\sqrt{\frac{2\,m\,V}{q}}\;N^{1/2} \qquad (6.9)$$

6.7 CYCLOTRONS

$$E_{max} = \frac{1}{2}\frac{q^2 B^2 R^2}{m}$$

6.7.4 Theory

From Eq. (6.8), it is clear that in order to accelerate particles to higher energy, we can

- increase ↑ ***B***, i.e. make magnets with higher and higher pole strength. But increasing *B* beyond a certain limit is not possible as most of the **materials saturate at a particular magnetic field.**
- increase ↑ ***R*** i.e. build pole pieces of larger and larger diameter so that the particles are extracted at larger R.

6.7 CYCLOTRONS

Advantages

1. Cyclotron is much **smaller in size** compared to linear accelerators.

2. No high voltages (like in Van de Graaff accelerator) are required. Only low-voltage ac oscillator (10-50 kV peak value) is required.

3. Cyclotron can deliver tens of microamperes of current at the target.

6.7 CYCLOTRONS

6.7.6 Limitations (1) Cost

- It has been estimated that the cost of building larger cyclotrons scales roughly as the cube of the energy.

- For example, the cost of **500 MeV** cyclotron is about **USS 108**.
- To build a cyclotron of **5 GeV** is beyond the means of most of the countries.

6.7 CYCLOTRONS

6.7.6 Limitations (2)

- As the energy of ions increases, **relativistic effects** come into picture.
- Because of the relativistic effects, mass of the ions m increases. m was assumed to be constant in Eqs. (6.4) and (6.5). Now it becomes a function of the velocity of the ions. This causes the ions to go out of phase with the applied ac oscillations and ions instead of accelerating may start de-accelerating.
- The upper limit of energy in case of protons is about **25-40 MeV.**

6.7 CYCLOTRONS

6.7.6 Limitations (2)

- At **higher energies**, mass of the ion increases due to relativistic effects to such an extent that its effect becomes visible.

- From Eq. (6.5), it is clear that as the mass of the ion increases, its **time of transient** through the dees also increases.
- As a result, the ion no longer are in phase with the oscillating potential. Instead of reaching the gap between the dees at the exact instant required for them to receive acceleration, the ions arrive too late and consequently gain little or no additional energy.

6.7 CYCLOTRONS

6.7.6 Limitations (2)

- On account of the relativistic mass increase, therefore an approximate limit sets on the energy that can be acquired by an ion in a conventional cyclotron operating under given conditions.

6.7 CYCLOTRONS

6.7.6 Limitations (2)

- It was shown that allowance could be made for the effect of the increase of mass of the ions moving at high speed so as to keep them in phase with the oscillating potential. Two methods for this compensation are possible

6.7 CYCLOTRONS

6.7.6 Limitations (2)

- One method is to leave the magnetic field unchanged, but to decrease the frequency of the oscillating potential in accordance with Eq. (6.6). As the mass of the particle increases fm is kept constant. Cyclotrons based on this principle are known as Synchrocyclotrons or Frequency Modulated Cyclotron,

6.7.6 Limitations (2)

- The other method is to increase the magnetic field B in proportion to the mass, so that B/m remains constant. Due to this, the time period / in Eg. (6.5) remains unaffected, by the increase of mass.
- Accelerators based on this principle are known as Sector Focusing or ***Azimuthally Varying Field* (AVF) Cyclotrons.**

Thanks