**Fourier Series:**

Fourier series for the function f(x) in the interval α<x<x+2πis given by

These equations for and are known as Euler’s formula.

**Example: Obtain the Fourier series for in the interval 0<x<.**

The fourier series for is given by

(1)

(3)

This is a standard integral

Comparing Eqn.4 with standard form we have a=-1 and b=n

Using the identities

Now evaluating a1, a2, a3 by substituting n=1,2,3… respectively in Eqn. (5)

we have

Now evaluating (6)

The standard integral is (7)

Comparing Eqn. (6) with standard integral of Eqn. (7) we get, a=-a and b=n

Now evaluating b1, b2 b3…from above equation keeping n=1, 2, 3… etc. we get

Combining all the results we write the fourier series for as

**Example: Obtain the Fourier series for in the interval -<x<.**

The fourier series for is given by

(1)

We know that

(3)

(4)

This is a standard integral

Comparing Eqn.4 with standard form we have a=-a and b=n

Using the identities

(5)

Now evaluating a1, a2, a3 by substituting n=1,2,3… respectively in Eqn. (5)

we have

Now evaluating (6)

The standard integral is (7)

Comparing Eqn. (6) with standard integral of Eqn. (7) we get, a=-a and b=n

Using the identities

Now evaluating b1, b2, b3…from above equation keeping n=1, 2, 3… etc. we get

Combining all the results we write the fourier series for as

**Example: Prove that in the interval** -

Fourier series is given by

Using the standard method

Considering, We get

Using the identities

Now we evaluate *bn*using formula

Using the standard method

Considering, We get

Now we use the identities

Combining all results and substituting the values of a0, an and bn we get

Exercise

**Find a0 for the function f(x) = x+x2 in the Fourier series for the interval - π < x< π**

1. **Find a0 for the function f(x) = xsinx in the fourier series for the interval - π < x< π**

;

3. Find a0 for the Fourier series to represent x2 in the interval (-π to π)

4. **Find a0 for f(x)=xsinx in the limit 0 to 2π**

We know that cos2; Hence

**Functions having points of discontinuity**

If the interval f(x) is defined as

Then c is called point of discontinuity. So we redefine a0, an and bn as

**Example: Find the Fourier series expansion for if**

Fourier series is given by

**Using the identities sinnπ=0,sin0=0, cosnπ= cos0=1**

Combining all the results we get

Putting x=0 in above equation all sinx terms will vanish as sin0=0

Now keeping x=0 in LHS terms of Eqn.1 we get

**Hence Proved**

Example: Find the Fourier series to represent the function f(x) given by

Deduce that

Fourier series is given by

Using the identities;

All sine terms vanishes in the above equation and hence we get

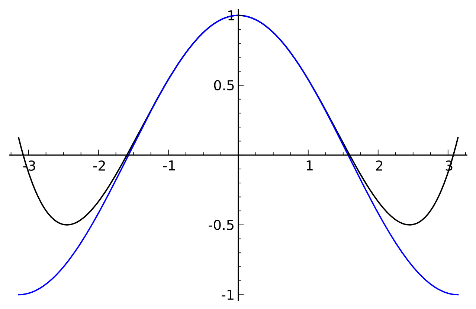
All sine term vanishes and cos terms cancel

Combining all results we get

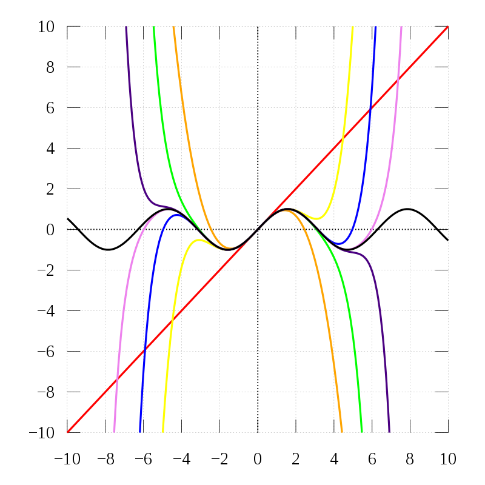
Substituting x=0 in LHS and RHS of equations we get

**Even and Odd functions:**

A function f(x) is said to be an Even if f(-x)=f(x). e.g. cosx, secx, x2. Graphically an even function is symmetric about Y-axis.



A function f(x) is said to be an Odd if f(-x)=-f(x). e.g. sinx, tanx, x3. Graphically an Odd function is symmetric about Origin.



**Fourier series expansion for Even function:**

Fourier series f(x) in the interval (-C, C) for an even function is defined as

And *bn*term will vanish since sinnπx = 0.

**Fourier series expansion for Odd function:**

In case of Odd function both and will vanish. Only exists which is given by

Example: Express as a fourier series in the interval –π <x < π.

As

So both and

**Hence**

**Find a fourier series to represent x2 in the interval (*-*).**

As ; So is an even function. So only and exists.

Now

Sinnπ=0 and hence first and last term on R.H.S. vanishes.

Hence putting the values of the series becomes

**Assignment Unit III**

**Multiple choice questions:**

The fourier series for f(x) in the interval  is given by



1. Cos n=
2. –n

(ii) (-1)n

1. 0

(iv) 1

1. Sin n=
2. –n

(ii) (-1)n

1. 0

(iv) 1

1. Sin(-) =
2. - sin 
3. sin 
4. None of above
5. Cos(-) =

(i) - cos 

(ii) cos 

(iii) None of above

6. A function f(x) is even if f(-x)

(i) = -f(x)

(ii) = f(x)

(iii) = 0

7. A function f(x) is said to be odd if f(-x)

(i) =f(x)

(ii) =-f(x)

(iii) =0

8. Examples of even functions are

(i) tan x

(ii) x3

(iii) sec x

9. Examples of odd function are

(i) x2

(ii) cos x

(iii) tan x

10. For odd function,

(i) a0 will vanish

(ii) an will vanish

(iii) bn will vanish

11. For even function,

(i) a0 will vanish

(ii) an will vanish

(iii) bn will vanish

12. 



13. Even function is symmetrical about

(i) X-axis

(ii) Y- axis

(iii) Origin

14. Odd function is symmetrical about

(i) X-axis

(ii) Y- axis

(iii) Origin

**Q-2 Short Questions:**

1. Find a0 for the function f(x) = x+x2 in the fourier series for the interval - π < x< π

2. Find a0 for the function f(x) = xsinx in the fourier series for the interval - π < x< π

3. Find a0 for the Fourier series to represent x2 in the interval (-π to π)

4. Give expressions for ao, an and bn.

5. Define Even function.

6. Define Odd function.

7. Differentiate Even and Odd functions.

8. Express as a fourier series in the interval –π <x < π.  
9. Find a fourier series to represent x2 in the interval (*-*).

**Long Questions:** 1. Prove that 

**2.** Find the fourier series expansion of f(x) = e-ax in the interval - π < x < π.

3. Find the fourier series expansion of f(x) = e-x in the interval 0< x < 2π

4. Find the fourier series expansion for f(x) if

f(x) = - π : - π < x< π ,

= x ; 0 < x < π

Deduce that 

1. Find the fourier series expansion for f(x) if

f(x) = - π : 0 < x< π ,

= 2 π - x ; π < x < 2π

Deduce that 