

PHYSICS
USO1CPHY21
UNIT-3
Network Analysis and Bridges.
Part -1 Network Analysis/

Introduction:

In many practical circuits in electrical engineering there occur situation where a circuit element is in series or parallel combinations with the other elements. When certain theorems are applied to such electric network, either they simplify the network or render their analytical solution very easy. A network is said to solved or analyzed when all the voltages and currents in its different arms are determined.

There are two general approaches to network analysis.

(1) Direct Method:

Here the network is left in its original form to determine its different voltages and currents. Such methods are usually restricted to fairly simple circuits and includes Kirchhoff's law, Loop analysis, Nodal analysis, Superposition theorems etc.

(2) Network Reduction Method:

Here the original network is converted in to a much simpler equivalent circuit for rapid calculations of different quantities. This method can be applied to simple as well as complex network. Examples of this method are Thevenin's theorem and Norton's theorem.

Kirchhoff's Law:

Kirchhoff's laws are useful in determining the equivalent resistance of complicated network of conductors and for calculating the currents flowing in various conductors.

There are two laws,

(1) Kirchhoff's Current Law:

In any electric network, the algebraic sum of the currents meeting at a point is Zero. ($\sum i = 0$)

(2) Kirchhoff's Voltage Law:

At any instance of time, sum of the voltages in closed circuit is zero. ($\sum v = 0$)

1. NETWORK TERMINOLOGY:

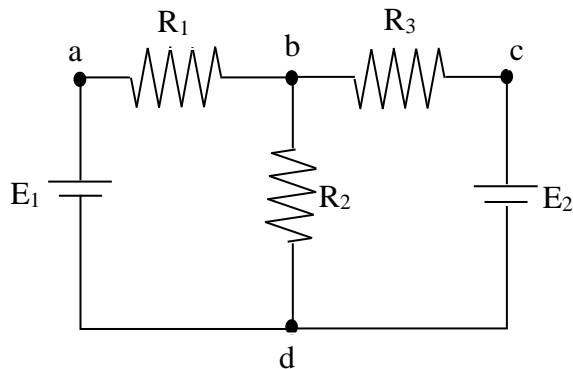


Fig. 1

3. **Junction:** A junction is that point in a network where three or more circuit elements are joined.

In the above network b and d are two junction points.

3. **Branch:** It is that part of a network which lies between junction points.

In the above network bad, bd and bcd are the branches.

4. **Loop:** Any closed path of a network is known as loop.

In Fig. 1 abda, bcdb, abcda are loops.

5. **Mesh:** It is the most elementary form of the loop which cannot be further divided into other loops.

In Fig. 1 abda and bcd are meshes but abcda is not a mesh because it encloses the first two loops.

6. **Mesh Current:** A mesh current is an imaginary current which flows around a mesh in a clockwise direction.

In Fig. 1, I_1 is the mesh current of mesh abda and I_2 is the mesh current of mesh bcd.

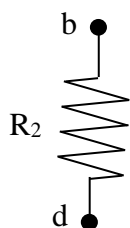


Fig. 2

8. **Tree:** A tree is defined as that part of a network which composed of those branches between the junction points which can be drawn without forming a closed path. In the network of Fig.1 bd forms a tree as shown in Fig 2.

The following terms are useful in analysis of electrical networks.

1. **Network:** It is an arrangement of passive and/or active elements which form closed paths.

In Fig.1 abcda is a network.

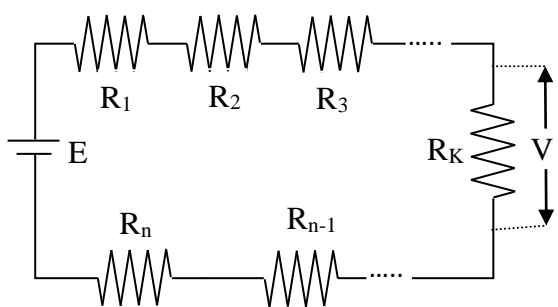
2. **Node:** It is a point in a network where two or more circuit elements are joined together.

In Fig. 1 a, b, c and d are node points.

2. VOLTAGE DIVIDER THEOREM:

This theorem is useful to determine voltage drop across a resistor in a series circuit like one as shown in Fig.3.

Statement: If n numbers of resistors are connected in series with source of emf



E, the voltage drops V_k across the terminals of any resistor R_k of them is given by,

$$V_k = \frac{R_k}{R_1 + R_2 + \dots + R_k + \dots + R_n} \times E$$

Fig.3

This is a voltage divider theorem.

Example 1: Using voltage divider theorem find V in circuit of Fig. 4.

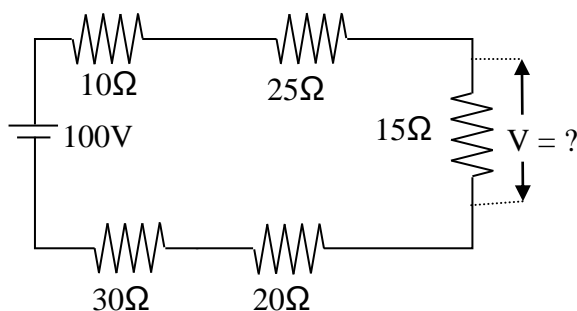


Fig.4

Example 2: Find voltage across resistor of 20Ω in the following circuit of Fig.5.

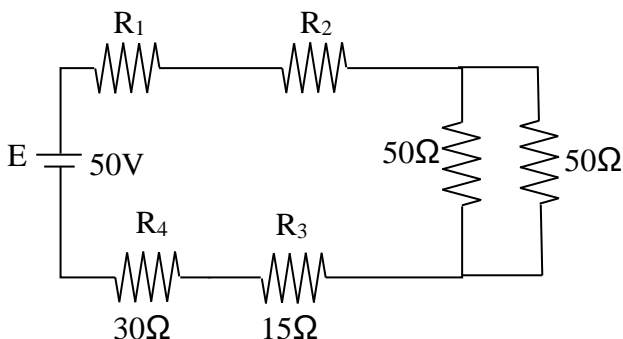


Fig. 5

Solution: The equivalent resistance of 50Ω and 50Ω resistors in parallel is

$$\frac{1}{R_p} = \frac{1}{50} + \frac{1}{50}$$

$$\therefore R_p = \frac{50 \times 50}{50+50} = \frac{2500}{100} = 25 \Omega$$

This R_p is now in series with other resistors. The voltage across 20Ω resistor is then given by,

$$V = \frac{R_2}{R_1 + R_2 + R_p + R_3 + R_4} \times E = \frac{20}{10 + 20 + 25 + 15 + 30} \times 50$$

$$\therefore V = \frac{20 \times 50}{100} = 10 \text{ V.}$$

3. SUPERPOSITION THEOREM:

Sometimes we need to find current through or voltage across a particular resistor in a series or linear circuit with two or even more than two sources of emf present in the circuit. The superposition theorem is useful in such cases.

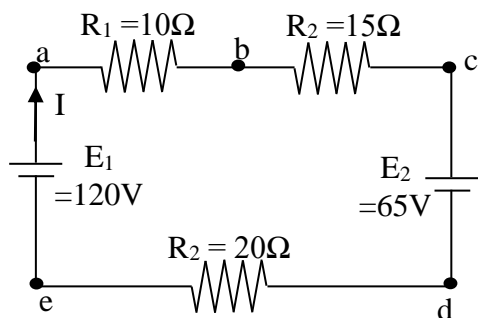


Fig. 6

Explanation:

Consider the circuit shown in Fig.6. It is a series circuit with two batteries (active elements) in series with three resistors (passive elements). Here we assume that loop current I flows in the clockwise direction as shown in the Fig. 6.

To find current I in the circuit we apply the Kirchoff's voltage law to the loop abcdea which gives,

$$IR_1 + IR_2 + IR_3 + E_2 - E_1 = 0$$

$$\therefore I(R_1 + R_2 + R_3) = E_1 - E_2$$

$$\therefore I = \frac{E_1 - E_2}{R_1 + R_2 + R_3} \dots (i)$$

$$\therefore I = \frac{E_1}{R_1 + R_2 + R_3} - \frac{E_2}{R_1 + R_2 + R_3} \dots (ii)$$

The first term in equation (ii) i.e., $\frac{E_1}{R_1 + R_2 + R_3}$ is the current (say I_1) in the circuit due to battery E_1 only i.e. when E_2 is removed and replaced by short circuit between points c and d. The current will flow in clockwise direction.

The second term in equation (ii) i.e., $\frac{E_2}{R_1 + R_2 + R_3}$ is the current (say I_2) in the circuit due to battery E_2 only i.e. when E_1 is removed and replaced by short circuit between points a and e. But in this case direction of current will be anticlockwise i.e. opposite to that due to E_1 .

When both E_1 and E_2 are present these two currents superimpose in the circuit to give the net or resultant current I . The current I is linearly related to voltage (emf). Following is the statement of superposition theorem.

Statement: If I_1 is the response to E_1 and I_2 is the response to E_2 then, in any linear circuit, the response to the combined function ($E_1 + E_2$) is ($I_1 + I_2$).

Example 3: Find the current I flowing in the circuit of Fig.6

For the circuit of Fig.6, Let I_1 be the current due to E_1 only. Then

$$I_1 = \frac{E_1}{R_1 + R_2 + R_3} = \frac{120}{45} = 2.66 \text{ A.} \quad \text{Here } I_1 \text{ will flow in clockwise direction.}$$

Similarly, Let I_2 be the current due to E_2 only. Then

$$I_2 = \frac{E_2}{R_1 + R_2 + R_3} = \frac{65}{45} = 1.44 \text{ A}.$$

Here I_2 will flow in anticlockwise direction i.e. opposite to I_1 .

According to superposition principle when both E_1 and E_2 are present, the resultant current I will be

$$I = I_1 + I_2$$

Since I_2 is opposite to I_1

We get
$$I = I_1 + (-I_2)$$

Hence
$$I = I_1 - I_2 = 2.66 - 1.44 = 1.22 \text{ A}.$$

Note that I will be in clockwise direction since $E_1 > E_2$.

Also I can be obtained directly by substituting the given values in equation (i),

$$\therefore I = \frac{120 - 65}{10 + 15 + 20} = \frac{55}{45} = 1.22 \text{ A}$$

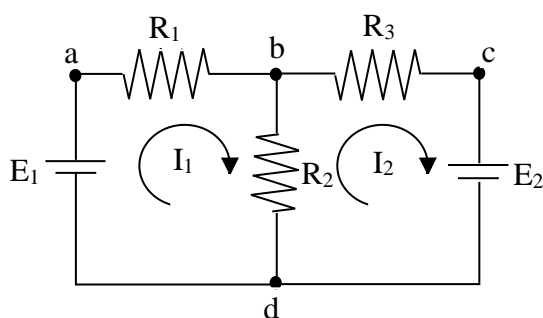
4. NETWORK ANALYSIS METHODS:

There are two main methods for network analysis namely;

(i) Mesh Current Method and (ii) Node-pair Voltage or Nodal Method.

4.1 Network Analysis by Mesh Current Method

4.1.1 Two - mesh Network:



A network consists of three resistors R_1 , R_2 , R_3 and two batteries E_1 and E_2 is shown in Fig.7. It consists of two meshes namely $abda$ and $bcdb$. Let I_1 be the mesh current of mesh $abda$ and I_2 be the mesh current of mesh $bcdb$ as shown. Also b and d are the two junction points and bad , bd and bcd are the three branches.

Fig. 7

To analyze the circuit using mesh current method, the number of independent mesh equation required m , is given by

$$m = b - (j - 1)$$

Where, b = number of branches in the network and

j = number of junction points in the network

In the network of Figure-1, we have $b = 3$ and $j = 1$.

$$m = 3 - (2 - 1) = 2.$$

Hence two independent mesh equations are required which can be obtained as follow:

For mesh-1 i.e. abda:

Applying Kirchoff's voltage to the mesh abda we get,

$$I_1 R_1 + I_1 R_2 - I_2 R_2 - E_1 = 0$$

$$\therefore I_1 (R_1 + R_2) - I_2 R_2 = E_1$$

But $R_1 + R_2 = R_{11} =$ self or total resistance of mesh-1 and

$R_2 = R_{12} =$ mutual resistance of mesh-1 and mesh-2.

Using these, above equation can be written as,

$$I_1 R_{11} - I_2 R_{12} = E_1 \quad \dots (1)$$

For mesh-2 i.e. bcdb:

Applying Kirchoff's voltage to the mesh bcdb we get,

$$I_2 R_3 + E_2 + I_2 R_2 - I_1 R_2 = 0$$

$$\therefore -I_1 R_2 + I_2 (R_2 + R_3) = -E_2$$

But $R_2 + R_3 = R_{22} =$ Self or total resistance of mesh-2 and

$R_2 = R_{21} =$ mutual resistance of mesh-2 and mesh-1.

Using these, above equation can be written as,

$$-I_1 R_{21} + I_2 R_{22} = -E_2 \quad \dots (2)$$

Equation (1) and (2) are the two requires equations to analyze the circuit using mesh current method. These equations can be solved for I_1 and I_2 if the values of resistances and batteries are known as explain in following example.

Example 4: Find current through R_2 i.e. through branch bd in the circuit of Fig.8

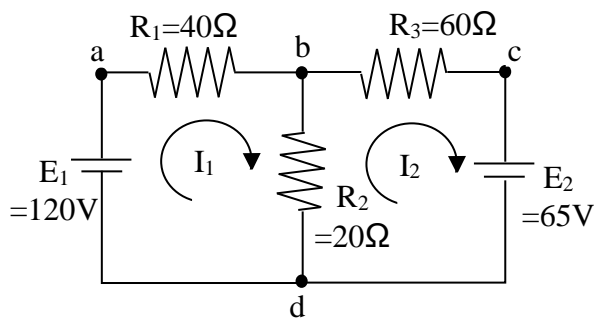


Fig. 8

For mesh-1 i.e. abda:

Applying Kirchoff's voltage to the mesh abda we get,

$$I_1 R_1 + I_1 R_2 - I_2 R_2 - E_1 = 0$$

$$\therefore I_1 (R_1 + R_2) - I_2 R_2 = E_1$$

Substituting values of resistances and battery we get

$$I_1 (40 + 20) - I_2 20 = 120$$

$$\therefore 60I_1 - 20I_2 = 120 \quad \dots(3)$$

For mesh-2 i.e. bcdb:

Applying Kirchoff's voltage to the mesh bcdb we get,

$$I_2 R_3 + E_2 + I_2 R_2 - I_1 R_2 = 0$$

Substituting values of resistances and battery we get

$$- 20I_1 + 80I_2 = - 65 \dots (4)$$

Multiplying equation (4) by and 3 and adding to equation (3) we get,

$$- 60I_1 + 240I_2 = - 195$$

$$60I_1 - 20I_2 = 120$$

$$220 I_2 = - 75$$

$$\therefore I_2 = - \frac{75}{220} = -0.34 \text{ A.}$$

Using this value of I_2 in equation (4) we get,

$$- 20I_1 + 80(-0.34) = - 65$$

$$- 20I_1 + (-27.2) = - 65$$

$$- 20I_1 = - 65 + 27.2 = - 37.8$$

$$\therefore I_1 = \frac{37.8}{20} = 1.89 \text{ A.}$$

The current through branch bd i.e. through resistor R_2 is, $I = I_1 + I_2$.

But, since I_2 is opposite to I_1 ,

$$I = I_1 + (- I_2) = I_1 - I_2$$

$$\therefore I = 1.89 + 0.34 = 2.23 \text{ A.}$$

This is the through R_2 i.e. through branch bd.

4.1.2 Three-mesh Network:

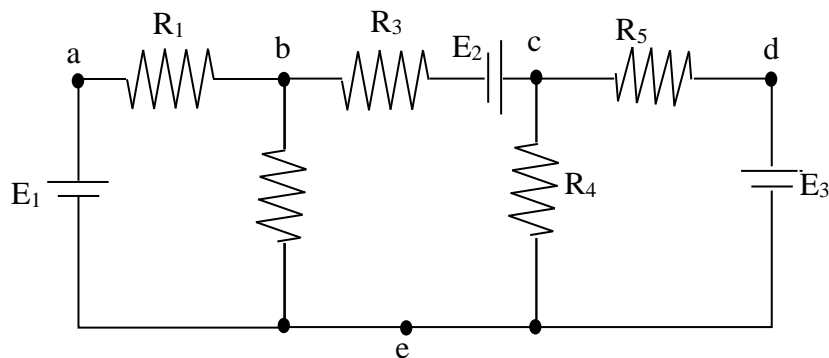


Figure 9.

A network consists of five resistors R_1, R_2, R_3, R_4, R_5 and three batteries E_1, E_2 and E_3 is shown in Fig. 9. In the network b, c and e are the three junction points. The network consists of three meshes namely abea, bceb and cdec. Let I_1 be the mesh current of mesh abea, I_2 be the mesh current of mesh bceb and I_2 be the mesh current of mesh cdec as shown. To analyze the circuit using mesh current method, the number of independent mesh equation required m , is given by

$$m = b - (j - 1)$$

Where, b = number of branches in the network and j = number of junction points in the network.

In Figure-2, we have 5 branches namely be, bae, bce, ce and cd. So we have $b = 5$. Also we have three junction points namely b, c and e. So $j = 3$.

$$\therefore m = 5 - (3 - 1) = 3.$$

Hence three independent mesh equation are required which can be obtained in a similar manner as we followed in case of two- mesh network.

For **mesh-1** i.e. **abea**:

Applying Kirchhoff's voltage we get,

$$I_1 (R_1 + R_2) - I_2 R_2 = E_1$$

But $R_1 + R_2 = R_{11} =$ self or total resistance of mesh-1 and

$$R_2 = R_{12} =$$
 mutual resistance of mesh-1 and mesh-2.

Using these, above equation can be written as,

$$I_1 R_{11} - I_2 R_{12} = E_1 \quad \dots(i)$$

For **mesh-2** i.e. **bceb**:

Applying Kirchhoff's voltage we get,

$$- I_1 R_2 + I_2 (R_2 + R_3 + R_4) - I_3 R_4 = E_2$$

$$\therefore - I_1 R_{21} + I_2 R_{22} - I_3 R_{23} = E_2 \quad \dots(ii)$$

where $R_{21} =$ mutual resistance of mesh-2 and mesh-1.

$R_{22} =$ Self or total resistance of mesh-2.

$R_{23} =$ mutual resistance of mesh-2 and mesh-3.

For **mesh-3** i.e. **cdcc**: Applying Kirchhoff's voltage we get,

$$\therefore - I_2 R_3 + I_3 (R_4 + R_5) = - E_3$$

$$\therefore - I_2 R_{32} + I_3 R_{33} = - E_3 \quad \dots(iii)$$

where $R_{32} =$ mutual resistance of mesh-3 and mesh-3.

$R_{33} =$ Self or total resistance of mesh-3.

The equation (i), (ii) and (iii) can be solved to determine I_1 , I_2 and I_3 .

4.2 Network Analysis by Node-pair Voltage or Nodal Method.

4.2.1 One-node pair network.

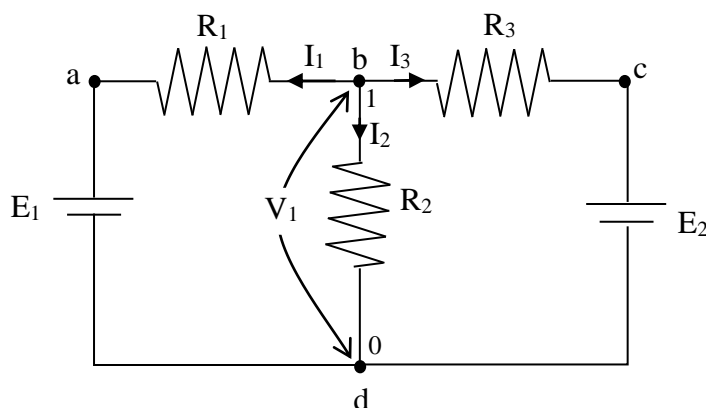


Fig. 10

Let the node potential between these points be V_1 as shown in the figure. Also assume that V_1 is the highest potential in the network so that branch currents I_1 , I_2 and I_3 are leaving the points 1 as shown. To analyze the circuit using node voltage method, the number of independent node equation required n , is given by,

$$n = j - 1.$$

Where, j = number of junction points in the network.

Since we have $j = 2$, we get $n = 2 - 1 = 1$.

Hence one nodal equation is required which can be obtained as follows.

Applying Kirchoff's Current law to the node point 1 we get,

$$I_1 + I_2 + I_3 = 0 \quad \dots(i)$$

Applying KVL to branch bad we get, $V_1 = I_1 R_1 + E_1$

$$\therefore I_1 = \frac{V_1 - E_1}{R_1} \quad \dots(ii)$$

Applying KVL to branch bd we get, $V_1 = I_2 R_2$

$$\therefore I_2 = \frac{V_1}{R_2} \quad \dots(iii)$$

Applying KVL to branch bcd we get, $V_1 = I_3 R_3 + E_2$

$$\therefore I_3 = \frac{V_1 - E_2}{R_3} \quad \dots(iv)$$

Using equation (ii), (iii) and (iv) in (i) we get

$$\frac{V_1 - E_1}{R_1} + \frac{V_1}{R_2} + \frac{V_1 - E_2}{R_3} = 0$$

$$\therefore V_1 \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \right) = \frac{E_1}{R_1} + \frac{E_2}{R_3} \quad \dots(v)$$

Here $\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \right)$ is the self conductance of node 1 = G_{11} .

$$\therefore V_1 G_{11} = \frac{E_1}{R_1} + \frac{E_2}{R_3} \quad \text{This is the required equation.}$$

Example 5: In the network of Fig.11 find I_2 through R_2 .

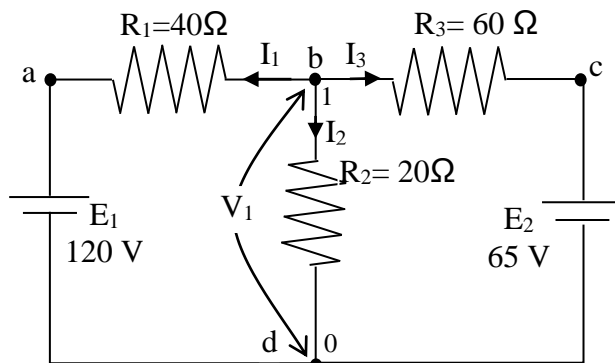


Fig.11

Using equation (v), we get

$$V_1 \left(\frac{1}{40} + \frac{1}{20} + \frac{1}{60} \right) = \frac{120}{40} + \frac{65}{60}$$

$$\therefore V_1 (0.091) = 4.08$$

$$\therefore V_1 = 44.83 \text{ V}$$

$$\text{Since } I_2 = \frac{V_1}{R_2}, \quad \text{we get } I_2 = \frac{44.83}{20} = 2.24 \text{ A.}$$

4.2.2 Two-node pair network.

A network consists of three resistors R_1, R_2, R_3, R_4, R_5 and three batteries E_1, E_2 and E_3 is shown in Fig. 12. As shown b, c and e are the three junction points as well as node points. The pair of points 1(or b) and 0(or e) forms a node-1 in the network and the pair of points 2(or c) and 0(or e) forms a node-2 in the network. Let the potential of node-1 be V_1 and that of node-2 be V_2 as shown in the Fig. 12.

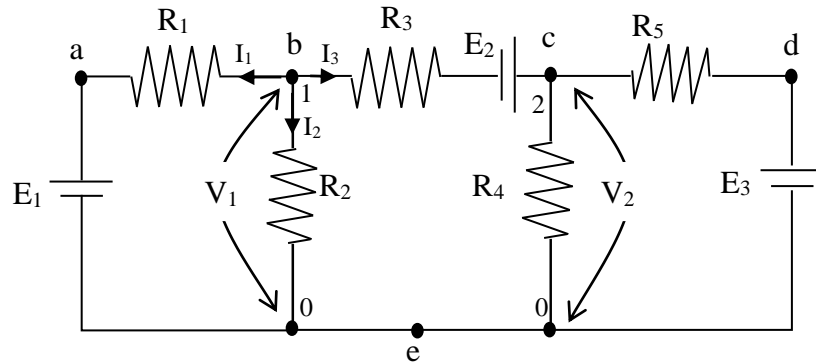


Fig. 12

To analyze the circuit using node voltage method, the number of independent node equation required n , is given by, $n = (j - 1)$ where, $j =$ number of junction points in the network.

Since we have $j = 3$, we get $n = (3 - 1) = 2$.

Hence, two nodal equations are required which can be obtained as follows.

For node-1:

We assume that V_1 is the highest potential in the network so that branch currents I_1, I_2 and I_3 are leaving the point 1 as shown in Figure-4. Applying Kirchhoff's Current law to the node point 1 we get,

$$I_1 + I_2 + I_3 = 0 \quad \dots (i)$$

Applying KVL to branch bae we get, $V_1 = I_1 R_1 + E_1$

$$\therefore I_1 = \frac{V_1 - E_1}{R_1} \quad \dots (ii)$$

Applying KVL to branch be we get, $V_1 = I_2 R_2$

$$\therefore I_2 = \frac{V_1}{R_2} \quad \dots (iii)$$

Applying KVL to branch cde we get, $V_1 = I_3 R_3 - E_2 + V_2$

$$\therefore I_3 = \frac{V_1 - V_2 + E_2}{R_3} \quad \dots (iv)$$

Using equation (ii), (iii) and (iv) in (i) we get

$$\frac{V_1 - E_1}{R_1} + \frac{V_1}{R_2} + \frac{V_1 - V_2 + E_2}{R_3} = 0.$$

$$\therefore V_1 \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \right) - V_2 \left(\frac{1}{R_3} \right) = \frac{E_1}{R_1} - \frac{E_2}{R_3} \quad \dots (v)$$

Here $\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \right)$ is the self conductance of node 1 = G_{11} .

and $\left(\frac{1}{R_3} \right)$ is the mutual conductance of node 1 & 2 = G_{12} .

$$\therefore V_1 G_{11} - V_2 G_{12} = \frac{E_1}{R_1} + \frac{E_2}{R_3} \quad \dots (a)$$

This is the first required equation.

For node-2:

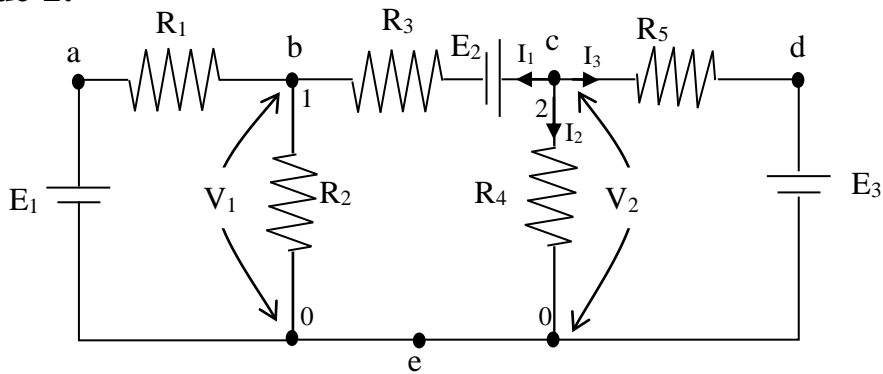


Fig. 13

In this case we assume that V_2 is the highest potential in the network so that current I_1 through branch cb, I_2 through branch ce and I_3 through branch cde are leaving the point 2(or c) as shown in Fig. 13.

Applying Kirchoff's Current law to the node point c we get,

$$I_1 + I_2 + I_3 = 0 \quad \dots (i)$$

Applying KVL to branch cb we get, $V_2 = E_2 + I_1 R_3 + V_1$

$$\therefore I_1 = \frac{V_2 - V_1 - E_2}{R_3} \quad \dots (ii)$$

Applying KVL to branch ce we get, $V_2 = I_2 R_4$

$$\therefore I_2 = \frac{V_2}{R_4} \quad \dots (iii)$$

Applying KVL to branch cde we get, $V_2 = I_3 R_5 + E_3$

$$\therefore I_3 = \frac{V_2 - E_3}{R_5} \quad \dots (iv)$$

Using equation (ii),(iii) and (iv) in (i) we get,

$$\frac{V_2 - V_1 - E_2}{R_3} + \frac{V_2}{R_4} + \frac{V_2 - E_3}{R_5} = 0$$

$$\therefore -V_1 \left(\frac{1}{R_3} \right) + V_2 \left(\frac{1}{R_3} + \frac{1}{R_4} + \frac{1}{R_5} \right) = \frac{E_2}{R_3} + \frac{E_3}{R_5} \quad \dots (v)$$

Here $\left(\frac{1}{R_3} + \frac{1}{R_4} + \frac{1}{R_5}\right)$ is the self conductance of node 2 = G_{22} .

and $\left(\frac{1}{R_3}\right)$ is the mutual conductance of node 2 & 1 = G_{21} .

$$\therefore -V_1 G_{21} + V_2 G_{22} = \frac{E_2}{R_3} + \frac{E_3}{R_5} \quad \dots (b)$$

The equation (a) and (b) are the two required equations.

5. Thevenin's Theorem

This theorem is useful to:

- (i) Convert a complicated network into a simple series circuit and
- (ii) Find branch current when branch resistance changes frequently.

Statement: Any two terminals of a network composed of active and passive circuit elements can be replaced by:

- (i) an equivalent voltage source, which is the voltage across that two terminals with all external elements disconnected and
- (ii) an equivalent series resistance, which is the equivalent resistance between that two terminals with all sources of emf shorted.

Consider network of Fig.14. As shown E_b is the emf, of battery, r_i is the internal resistance of the battery, S is a switch and R_L is the load resistance.

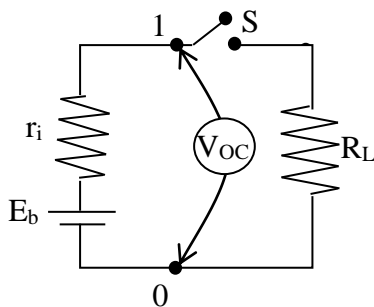
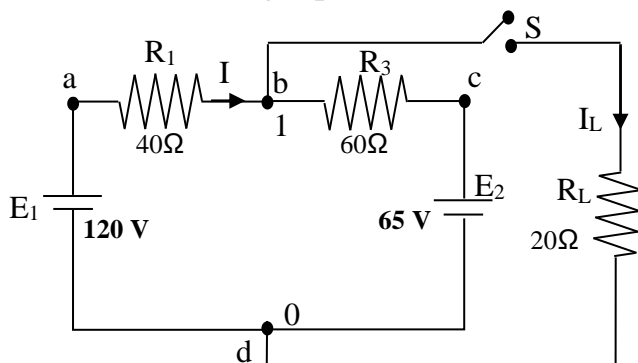


Fig. 14

Hence, once E_b and r_i are known, I_L can be determined easily for different values of R_L using equation (25).



Application/Example 6:

Suppose we want to find the I_L For different values of R_L in the network of Fig. 15. Then using Thevenin's theorem we can convert this complicated network into simple series circuit in following way.

Fig. 15

Step-1. Determine open circuit voltage V_{01} across terminals 1 and 0 (i.e. with R_L disconnected.)

For this let S be open in Figure 7. Now let I be the current through loop $abcd$. Then using superposition theorem we get,

$$I = \frac{E_1 - E_2}{R_1 + R_3} \dots (i).$$

Putting values for network of Figure 7,

$$I = \frac{120 - 65}{40 + 60} = \frac{55}{100} = 0.55 \text{ A.}$$

The open circuit voltage V_{01} is given as, $V_{01} = E_1 - IR_1 = E_2 + IR_3$.

$$\therefore V_{01} = 120 - (0.55)(40) = 120 - 22 = 98 \text{ V.}$$

This the value of open circuit voltage across terminal 1 and 0.

Step 2. Determine equivalent series resistance R_i between terminal 1 and 0 with all sources of emf shorted.

For this let's short terminal a with d and c with d. As a result of this R_2 and R_3 becomes parallel to each other as shown in Figure 8. The equivalent resistance R_i between 1 and 0 is then given by

$$r_i = \frac{R_1 R_3}{R_1 + R_3} \dots (ii)$$

Putting values for network of Figure 7, we get

$$r_i = \frac{40 \times 60}{40 + 60} = \frac{2400}{100} = 24 \Omega$$

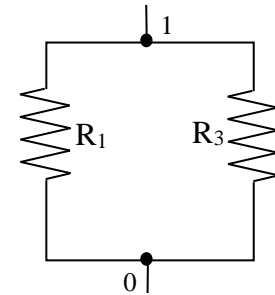


Fig.16

Step-3. Construction of final series circuit.

Using Thevenin's theorem, the circuit across terminals 1 and 0 of Fig.15 can be replaced as shown in Fig. 17, which is a simple series circuit.

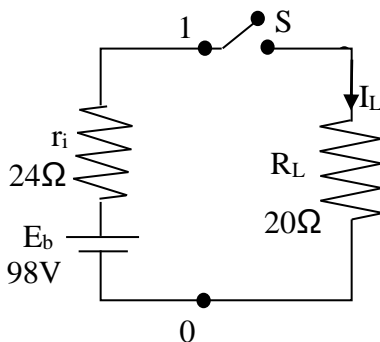


Fig.17

The current I_L is then,

$$I_L = \frac{E_b}{(r_i + R_L)}$$

$$\therefore I_L = \frac{98}{(24 + 20)} = 2.22 \text{ A.}$$

Example 7: Find I_L when $R_L = 40\Omega$, 55Ω , and 60Ω .

6. Norton's Theorem:

This theorem explains about the current equivalent of a voltage source. It was first developed by E.L. Norton at the Bell Telephone laboratory. This theorem is an alternative to the Thevenin's theorem. Whereas Thevenin's theorem reduces a two terminal active network of linear resistance and sources to an equivalent constant voltage source and series resistance, Norton's theorem replaces the network by an equivalent constant current source and a parallel resistance. Norton's theorem is very much useful for the circuit containing transistor and other devices which are current sensitive.

Statement:

“Any two terminals of a network composed of linear passive and active circuit elements may be replaced by (i) an equivalent current source and (ii) a parallel resistance. The current of the source is the current measured in the short circuit placed across the terminal pair. The parallel resistance is the equivalent resistance looking into the terminal pair with all independent power sources inactive.”

Explanation:

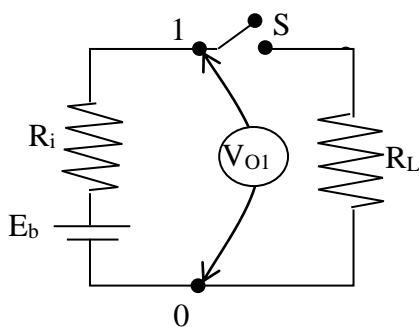


Fig.18

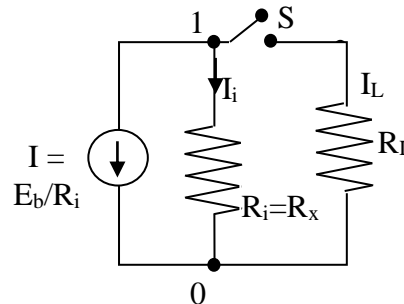


Fig. 19

Consider the circuit of Fig.18. When R_L is zero, a current flows through the output terminals from 1 to 0. This current is maximum possible current that can be supplied by battery or given voltage source. Hence we can consider the voltage source as a source of current.

Let I be the maximum current through R_i then we can write, $I = \frac{E_b}{R_i}$ (i).

Now this current source must provide the same voltage across terminal 1 and 0 of Fig.18 i.e. V_{01} . This can be done by connecting a shunt resistance R_x across the terminal 1 and 0 as shown in Fig. 19. Here Fig.19 shows the current source equivalent of a voltage source.

Now when S is open we get from Fig. 18, $I R_x = V_{01}$

Also from Fig.1. $V_{01} = E_b = I R_i$.

So we can write, $IR_x = V_{01} = E_b = IR_i \quad \therefore R_x = R_i.$

Hence as long as S is open, the left part of terminal 1 and 0 of Fig. 18 can be replaced by an equivalent current source of magnitude $I = E_b/R_i$ and a shunt resistance of value R_i .

Now applying KVL to Fig.18, when S is closed, we get,

$$E_b = I_L R_i + I_L R_L$$

$$\therefore E_b - I_L R_i = I_L R_L$$

$$\therefore \frac{E_b}{R_i} - I_L = \frac{V_{01}}{R_i}$$

$$\therefore E_b - I_L R_i = V_{01} \cdot (V_{01} = I_L R_L)$$

Since $E_b/R_i = I$, we get, $I - I_L = \frac{V_{01}}{R_i}$ (ii)

Using this expression, I_L can be determinate.

Applying KCL to node point 1 of Fig. 19 we get,

$$I = I_i + I_L \quad \therefore I = \frac{V_{01}}{R_i} + \frac{V_{01}}{R_L}$$

$$\therefore I = V_{01} \left(\frac{1}{R_i} + \frac{1}{R_L} \right) = \frac{V_{01}}{R_p}$$

$$\therefore V_{01} = I \times R_p \quad \dots \text{(iii)}$$

Here I is equivalent current and R_p is equivalent resistance. Using above expression terminal voltage can be determined.

Example 8:

Find current through R_L for $R_L = 11, 20$ and 30Ω in circuit of Fig. 20.

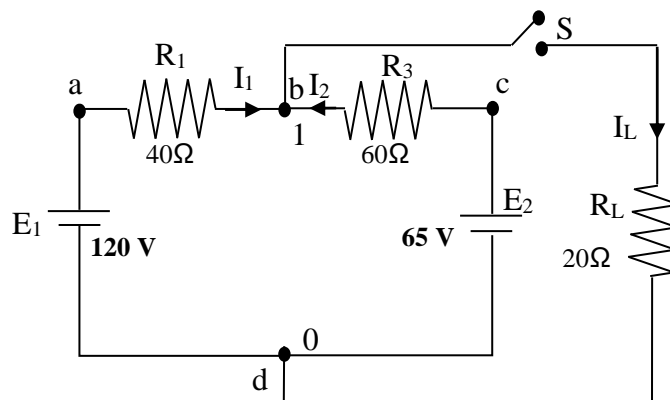


Fig. 20

From the diagram we have,

$$I_1 = \frac{E_1}{R_1} = \frac{120}{40} = 3.0 \text{ A} \quad \& \quad I_2 = \frac{E_2}{R_3} = \frac{60}{65} = 1.08 \text{ A} .$$

The total current through the short-circuit across terminals 1 & 0 is then,

$$I = I_1 + I_2 = 3.00 + 1.08 = 4.08 \text{ A} \quad \dots \text{ (i)}$$

Now, equivalent resistance across terminals 1 & 0 is,

$$R_i = R_1 \parallel R_3$$

$$\therefore R_i = \frac{R_1 R_3}{R_1 + R_3} = \frac{40 \times 60}{40 + 60} = \frac{2400}{100} = 24 \Omega \quad \dots \text{ (ii)}$$

Hence, using values in equation (i) & (ii) we get equivalent circuit of Fig. 20 as shown in Fig. 21.

Applying KCL to point 1 in Fig. 21 when S is closed, we get,

$$I = I_1 + I_2$$

$$\therefore I_1 = I - I_2$$

$$\text{Also, } I_1 R_1 = I_2 R_2$$

$$\therefore I_1 R_i = (I - I_2) R_i$$

Using these results, we can write,

$$I_2 R_2 = (I - I_2) R_i = I R_i - I_2 R_i$$

$$\therefore I_2 R_2 + I_2 R_i = I R_i$$

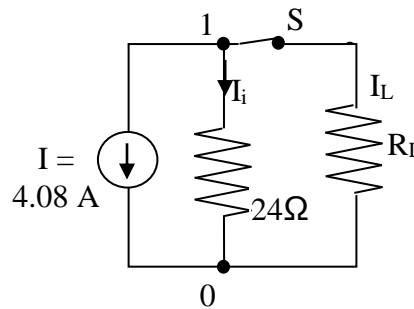
$$\therefore I_2 (R_2 + R_i) = I R_i$$

$$\therefore I_2 = \frac{R_i}{R_2 + R_i} I \quad \dots \text{ (iii)}$$

Using above relation we get.

$$I_2 = \frac{24}{24 + 11} \times 4.08 = 2.8 \text{ A}$$

Similarly we can find I_2 for $R_2 = 20 \ \& \ 30 \ \Omega$.



ASSIGNMENT

A Multiple Choice Questions:

1	A point in a network where three or more circuit elements are joined together is call as (a) Junction (b) node (c) branch (d) tree.
2	Which current always flows in a clockwise direction, (a) mesh current (b) loop current (c) branch current (d) node current.
3	How many number of independent node equations are required to analyze a network having three junction point and five branches? (a) 2 (b) 6 (c) 5 (d) 3
4	How many number of independent mesh equations are required to analyze a network having three junction point and five branches? (a) 2 (b) 6 (c) 5 (d) 3
5	When two resistors are connected in parallel, their equivalent resistance (a) increases (b) may not change (c) decreases (o) becomes zero.

B. Short answer questions.

1	State and explain voltage divider theorem.
2	What is superposition principle ? Give its statement.
3	Three resistors, $R_1=10\ \Omega$, $R_2=20\ \Omega$ and $R_3=30\ \Omega$ are connected in series with a batter of 10V. Find the voltage drop across R_2 .
4	Three resistors, $R_1=10\ \Omega$, $R_2=20\ \Omega$ and $R_3=30\ \Omega$ are connected in parallel with a batter of 10V. Find the current passing through R_2 .
5	State Thevenin's theorem.
6	State Norton theorem.

C. Long answer questions.

1	With proper example explain mesh current analysis of two mesh network.
2	Discuss nodal method of network analysis for a two node pair network.
3	Explain: Thevenin's theorem.
4	Using necessary network diagram explain network terminology.
5	With proper example explain nodal analysis of one node pair network.
6	With proper network explain Norton theorem.
7	Give a detailed comparison of Thevenin's and Norton's theorem.

EXCERSICE
MULTIPLE CHOICE QUESTIONS

1. Superposition theorem is useful in analyzing a network containing
(a) one voltage source (b) one resistor
(c) **more than one voltage sources** (d) more than one resistors
2. Superposition theorem is applicable to the networks in which the current and voltage has
(a) **linear relation** (b) exponential relation
(c) logarithmic relation (d) quadratic relation
3. For two resistors connected in series with each other, the following quantity will be the same
(a) voltage (b) **current** (c) frequency (d) amplitude
4. For two resistors connected in parallel with each other, the following quantity will be the same
(a) **voltage** (b) current (c) frequency (d) amplitude
5. Every junction is a _____. Complete the sentence choosing the proper word.
(a) **node** (b) mesh (c) loop (d) branch
6. Every node may not be a _____. Complete the sentence choosing the proper word.
(a) mesh (b) **junction** (c) loop (d) branch
7. Every mesh is a _____. Complete the sentence choosing the proper word.
(a) node (b) mesh (c) **loop** (d) branch
8. Every loop may not be a _____. Complete the sentence choosing the proper word.
(a) **mesh** (b) junction (c) loop (d) branch
9. Node is a point of a network where the number of components connected are
(a) one (b) two (c) **two or more** (d) one or more
10. Junction is a point of a network where the number of components connected are
(a) three (b) two (c) two or more (d) **three or more**
11. Mesh method of network analysis makes use of the following to analyze a network.
(a) **Kirchhoff's Voltage Law** (b) Kirchhoff's Current Law
(c) Faraday's Law (d) Lenz's Law
12. Node-Pair voltage method of network analysis makes use of the following to

analyze a network.

- (a) Kirchhoff's Voltage Law **(b) Kirchhoff's Current Law**
(c) Faraday's Law (d) Lenz's Law

13. The reciprocal of resistance is called

- (a) impedance (b) reactance **(c) conductance** (d) reluctance

14. In Thevenin's theorem V_{oc} refers to

- (a) open component voltage
(b) open controlled voltage
(c) open circuit voltage
(d) on component voltage

15. In Thevenin's theorem I_{sc} refers to

- (a) short circuit current**
(b) short component current
(c) single circuit current
(d) single component current